Package ‘BTYD’

February 19, 2015

Type Package
Title Implementing Buy ’Til You Die Models
Version 2.4
Date 2014-11-06
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URL wcai.wharton.upenn.edu
Description This package contains functions for data preparation, parameter estimation, scoring, and plotting for the BG/BB, BG/NBD and Pareto/NBD models.
License GPL-3
Collate 'bgbb.R' 'bgnbd.R' 'pnbd.R' 'dc.R' 'spend.R'
Depends hypergeo
Imports Matrix
Suggests knitr
VignetteBuilder knitr
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NeedsCompilation no
Repository CRAN
Date/Publication 2014-11-07 09:59:40

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Implementing Buy 'Til You Die Models

Description

This package contains functions for data preparation, parameter estimation, scoring, and plotting for the BG/BB, BG/NBD and Pareto/NBD models.
Details
This project was funded and sponsored by the Wharton Customer Analytics Initiative (wcai.wharton.upenn.edu).

This package implements the BG/BB, BG/NBD and Pareto/NBD models, which capture/project customer purchase patterns in a typical non-contractual setting.

While these models are developed on a customer-by-customer basis, they do not necessarily require data at such a granular level. The Pareto/NBD requires a “customer-by-sufficient-statistic” matrix (CBS), which consists of each customer’s frequency, recency (the time of their last transactions) and total time observed - but the timing of each and every transaction (other than the last) is not needed by the model. If, however, you do have the granular data in the form of an event log (which contains at least columns for customer identification and the time of each transaction, and potentially more columns such as transaction amount), this package provides functions to convert it to a CBS. You can use \texttt{dc.ReadingLines} to get your event log from a comma-delimited file to an event log usable by this package; it is possible to use read.table or read.csv, but formatting will be required afterwards. You can then convert the event log directly to a CBS (for both the calibration and holdout periods) using \texttt{dc.ElogToCbsCbt}. As the name suggests, this function also produces a customer-by-time matrix (CBT). This matrix consists of a row for every customer and a column for every date, and is populated by a statistic of your choice (reach, frequency, or spend). It is not necessary for any of the models presented in this package, but is used as a building block to produce the CBS.

The BG/NBD model requires all the same inputs as the Pareto/NBD model.

The BG/BB model requires the same information as the Pareto/NBD model, but as it models discrete transaction opportunities, this information can be condensed into a recency-frequency matrix. A recency-frequency matrix contains a row for every recency/frequency combination in the given time period, and each row contains the number of customers with that recency/frequency combination. Since frequency will always be less than or equal to recency, this matrix will contain \((n)(n-1)/2 + 1\) rows at most, with \(n\) as the number of transaction opportunities (of course, the maximum number of rows for pooled data - for customers with varying numbers of transaction opportunities - will be the sum of the above equation for each unique number of transaction opportunities). You can convert a CBS to recency-frequency matrices using \texttt{dc.MakeRFmatrixCal} and \texttt{dc.MakeRFmatrixHoldout}.

If you want to test the data contained in the package, or have data formatted as a customer-by-sufficient-statistic or recency-frequency matrix, a good starting place would be \texttt{pnbd.EstimateParameters}, \texttt{bgnbd.EstimateParameters}, or \texttt{bgbb.EstimateParameters}.

Following that, \texttt{pnbd.PlotFrequencyInCalibration}, \texttt{bgnbd.PlotFrequencyInCalibration} and \texttt{bgbb.PlotFrequencyInCalibration} will give a check that the model fits the data in-sample. Further plotting functions, comparing actual and expected results, are labelled “pnbd.Plot...”, “bgnbd.Plot...” and “bgbb.Plot...". The building blocks of these functions are also provided: \texttt{pnbd.LL}, \texttt{bgnbd.LL} \texttt{bgbb.LL}, \texttt{pnbd.pm}, \texttt{bgnbd.pm}, \texttt{bgbb.pm}, \texttt{pnbd.Expectation}, \texttt{bgnbd.Expectation}, \texttt{bgbb.Expectation}, \texttt{pnbd.ConditionalExpectedTransactions}, \texttt{bgnbd.ConditionalExpectedTransactions}, and \texttt{bgbb.ConditionalExpectedTransactions}

This package uses the following conventions:
The time period used to estimate the model parameters is called the *calibration period*. Users may be accustomed to this being called the estimation period, or simply being referred to as “in-sample”. Function parameter names generally follow this convention: for example, “n.cal” is used to refer to the number of transaction opportunities in the calibration period.

The time period used to validate model performance is called the *holdout period*. Users may be accustomed to this being called the validation period, or simply being referred to as “out-of-sample”. Function parameters relating to this time period are generally appended with “.star”. For example, n.star is used to refer to the number of transaction opportunities in the holdout period.

As described in the papers referenced below, the BG/BB, BG/NBD and Pareto/NBD models are generally concerned with repeat transactions, not total transactions. This means that a customer’s first transaction in the calibration period is usually not part of the data being modelled - this is due to the fact that a new customer generally does not show up “on the company’s radar” until after their first purchase has taken place. This means that the modal number of repeat purchases tends to be zero. If your data does not have a relatively large number of customers with zero transactions, but does have a relatively large number of customers with one transaction, and the estimation functions are struggling, the problem is most likely that you are including customers’ very first transactions. Some of the data-conversion functions have examples illustrating how to work with data that includes this very first transaction. Note that this does not apply to the holdout period; in the holdout period, we already know about the customer and take all of their previous transactions into account.

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**References**

See [www.brucehardie.com](http://www.brucehardie.com) for papers, notes, and datasets relating to applied probability models in marketing.


bgbb. Conditional Expected Transactions

**BG/BB Conditional Expected Transactions**

**Description**

Calculates the number of expected transactions in the holdout period, conditional on a customer’s behavior in the calibration period.

**Usage**

\[ \text{bgbb. ConditionalExpectedTransactions}(\text{params}, \text{n.cal}, \text{n.star}, \text{x}, \text{t.x}) \]

**Arguments**

- **params**: BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
- **n.cal**: number of transaction opportunities in the calibration period, or a vector of calibration period transaction opportunities.
- **n.star**: number of transaction opportunities in the holdout period, or a vector of holdout period transaction opportunities.
- **x**: number of repeat transactions the customer made in the calibration period, or a vector of calibration period transaction frequencies.
- **t.x**: recency - the last transaction opportunity in which this customer made a transaction, or a vector of recencies.

**Details**

\[ E(X(n, n+n^*) | \alpha, \beta, \gamma, \delta, x, t.x, n) \]. This function requires the holdout period to immediately follow the calibration period.

n.cal, n.star, x, and t.x may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they


are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

The number of transactions a customer is expected to make in the n.star transaction opportunities following the calibration period, conditional on their behavior during the calibration period.

References


Examples

```r
params <- c(1.20, 0.75, 0.66, 2.78)
# the number of transactions a customer is expected
# to make in the 10 transaction opportunities
# following the calibration period, which consisted
# of 6 transaction opportunities (during which they
# made 3 transactions, the last of which occurred
# in the 4th opportunity)
bgbb.conditionalExpectedTransactions(params, n.cal=6, n.star=10, x=3, t.x=4)

# We can also use vectors as input:
bgbb.conditionalExpectedTransactions(params, n.cal=6, n.star=1:10, x=3, t.x=4)
bgbb.conditionalExpectedTransactions(params, n.cal=6, n.star=10, x=1:4, t.x=4)
```

---

**bgbb.DERT**

*BG/BB Discounted Expected Residual Transactions*

**Description**

Computes the number of discounted expected residual transactions by a customer, conditional on their behavior in the calibration period.

**Usage**

```r
bgbb.DERT(params, x, t.x, n.cal, d)
bgbb.rf.matrix.DERT(params, rf.matrix, d)
```
Arguments

params BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

x number of repeat transactions a customer made in the calibration period, or a vector of calibration period transaction frequencies.

t.x recency - the last transaction opportunity in which a customer made a transaction, or a vector of recencies.

n.cal number of transaction opportunities in the calibration period, or a vector of calibration period transaction opportunities.

d discount rate.

r.f.matrix recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), and the number of transaction opportunities in the calibration period ("n.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.

Details

DERT(d \mid \alpha, \beta, \gamma, \delta, x, t.x, n). This is the present value of the expected future transaction stream for a customer with x transactions and a recency of t.x in n.cal transaction opportunities, discounted by a rate d.

x, t.x, and n.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.

Value

The present value of the expected future transaction stream for a particular customer.

References


See equation 14.

Examples

params <- c(1.20, 0.75, 0.66, 2.78)
# Compute DERT for a customer who made 3 transactions
# in the calibration period (consisting of 6 transaction
# opportunities), with the last transaction occurring
# during the 4th transaction opportunity, discounted at
bgbb.EstimateParameters

BG/BB Parameter Estimation

Description

Estimates parameters for the BG/BB model.

Usage

```r
bgbb.EstimateParameters(rf.matrix, par.start = c(1, 1, 1, 1),
                         max.param.value = 1000)
```

Arguments

- `rf.matrix` recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), number of transaction opportunities in the calibration period ("n.cal"), and the number of customers with this combination of recency, frequency and transaction opportunities in the calibration period ("custs"). Note that recency must be the time between the start of the calibration period and the customer's last transaction, not the time between the customer's last transaction and the end of the calibration period.

- `par.start` initial BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

- `max.param.value` the upper bound on parameters.
Details

The best-fitting parameters are determined using the `bgbb.rf.matrix.LL` function. The sum of the log-likelihood for each customer (for a set of parameters) is maximized in order to estimate parameters.

A set of starting parameters must be provided for this method. If no parameters are provided, (1,1,1,1) is used as a default. It may be useful to use starting values for parameters that represent your best guess of the heterogeneity in the transaction and dropout rates of customers. It may be necessary to run the estimation from multiple starting points to ensure that it converges. To compare the log-likelihoods of different parameters, use `bgbb.rf.matrix.LL`.

The lower bound on the parameters to be estimated is always zero, since BG/BB parameters cannot be negative. The upper bound can be set with the `max.param.value` parameter.

Value

Vector of estimated parameters.

See Also

`bgbb.rf.matrix.LL`

Examples

```r
data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix
# donationsSummary$rf.matrix already has appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.Expectation(rf.matrix, startingparams)
# log-likelihood of estimated parameters
bgbb.rf.matrix.LL(est.params, rf.matrix)
```

Description

Returns the number of transactions that a randomly chosen customer (for whom we have no prior information) is expected to make in the first n transaction opportunities.

Usage

`bgbb.Expectation(params, n)`
Arguments

params: BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
n: number of transaction opportunities.

Details

\[ E(X(n) \mid \alpha, \beta, \gamma, \delta) \]

Value

Mean of the BG/BB probability mass function.

References


Examples

params <- c(1.20, 0.75, 0.66, 2.78)
# Expected number of transactions that a randomly chosen customer
# will make in the first 10 transaction opportunities.
bgb.min.10(params, n=10)

# We can also compare expected transactions over time:
bgb.min.10(params, n=1:10)

---

**bgbb.heatmapHoldoutExpectedTrans**

*BG/BB Heatmap of Holdout Period Expected Transactions*

Description

Plots a heatmap based on the conditional expected holdout period frequency for each recency-frequency combination in the calibration period.

Usage

bgbb.heatmapHoldoutExpectedTrans(params, n.cal, n.star, xlab = "Recency", ylab = "Frequency", xticklab = NULL, title = "Heatmap of Conditional Expected Transactions")
Arguments

params: BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

n.cal: number of transaction opportunities in the calibration period.

n.star: number of transaction opportunities in the holdout period.

xlab: descriptive label for the x axis.

ylab: descriptive label for the y axis.

xticklab: vector containing a label for each tick mark on the x axis.

title: title placed on the top-center of the plot.

Details

\( E(X(n, n+n^*) | alpha, beta, gamma, delta, x, t.x, n) \). This function requires the holdout period to immediately follow the calibration period.

Value

A matrix containing the conditional expected transactions in the holdout period for each recency-frequency combination in the calibration period. The rows represent calibration period frequencies, and the columns represent calibration period recencies.

References


http://www.brucehardie.com/papers/020/

See Also

`bgbb.heatmapHoldoutExpectedTransactions`

Examples

data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix
# donationsSummary$rf.matrix already has appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)

# Plot a heatmap of conditional expected transactions in
# a holdout period of 5 transaction opportunities, given
# that the calibration period consisted of 6 transaction
# opportunities.
bgbb.HeatmapHoldoutExpectedTrans(est.params, n.cal=6, n.star=5)

---

**bgbb.LL**

**BG/BB Log-Likelihood**

**Description**
Calculates the log-likelihood of the BG/BB model.

**Usage**

bgbb.rf.matrix.LL(params, rf.matrix)

bgbb.LL(params, x, t.x, n.cal)

**Arguments**

- **params**: BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
- **rf.matrix**: recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), number of transaction opportunities in the calibration period ("n.cal"), and the number of customers with this combination of recency, frequency and transaction opportunities in the calibration period ("custs"). Note that recency must be the time between the start of the calibration period and the customer's last transaction, not the time between the customer's last transaction and the end of the calibration period.
- **x**: the number of repeat transactions made by the customer in the calibration period. Can also be a vector of frequencies - see details.
- **t.x**: recency - the transaction opportunity in which the customer made their last transaction. Can also be a vector of recencies - see details.
- **n.cal**: number of transaction opportunities in the calibration period. Can also be a vector of calibration period transaction opportunities - see details.

**Details**

x, t.x, and n.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.
Value

For bgbb.cbs. LL, the total log-likelihood of the provided data. For bgbb. LL, a vector of log-likelihoods as long as the longest input vector (x, t.x, or n.cal).

References


Examples

data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix
# donationsSummary$rf.matrix already has appropriate column names

params <- c(1.20, 0.75, 0.66, 2.78)
bgbb.rf.matrix.LL(params, rf.matrix)

# Returns the log likelihood of the parameters for a customer who
# made 3 transactions in a calibration period with 6 transaction opportunities,
# with the last transaction occurring during the 4th transaction opportunity.
bgbb.LL(params, x=3, t.x=4, n.cal=6)

# We can also give vectors as function parameters:
set.seed(7)
x <- sample(1:3, 10, replace = TRUE)
t.x <- sample(3:5, 10, replace = TRUE)
n.cal <- rep(5, 10)
bgbb.LL(params, x, t.x, n.cal)

---

**bgbb.PAlive**  
*BG/BB P(Alive)*

Description

Uses BG/BB model parameters and a customer’s past transaction behavior to return the probability that they will be alive in the transaction opportunity following the calibration period.

Usage

bgbb.PAlive(params, x, t.x, n.cal)
Arguments

params  
BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

x  
number of repeat transactions a customer made in the calibration period. May also be a vector of frequencies - see details.

t.x  
recency - the last transaction opportunity in which this customer made a transaction. May also be a vector of recencies - see details.

n.cal  
number of transaction opportunities in the calibration period. May also be a vector of calibration period transaction opportunities - see details.

Details

x, t.x, and n.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

P(alive at n+1 | alpha, beta, gamma, delta, x, t.x, n)

Value

Probability that the customer is alive at the (n+1)th transaction opportunity. If x, t.x, and/or n.cal has a length greater than one, then this will be a vector of probabilities (containing one element matching each element of the longest input vector).

References


Examples

params <- c(1.20, 0.75, 0.66, 2.78)

# The probability that a customer who made 3 transactions in # the calibration period (which consisted of 6 transaction # opportunities), with the last transaction occurring at the # 4th transaction opportunity, is alive at the 7th transaction # opportunity
bgbb.PAlive(params, x=3, t.x=4, n.cal=6)

# The input parameters may also be vectors:
bgbb.PAlive(params, x=1, t.x=1:6, n.cal=6)
**Description**

Plots and returns the estimated beta distribution Theta (customers’ propensities to drop out).

**Usage**

```r
bgbb.PlotDropoutRateHeterogeneity(params)
```

**Arguments**

- `params` : BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

**Details**

This returns the distribution of each customer’s geometric parameter that determines their lifetime (using the BG/BB assumption that a customer’s lifetime can be modeled with an geometric distribution).

**Value**

Distribution of customers’ propensities to drop out.

**Examples**

```r
params <- c(1.2, 0.75, 0.66, 2.78)
bgbbr.PlotDropoutRateHeterogeneity(params)
params <- c(0.2, 1.5, 3.2, 6)
bgbbr.PlotDropoutRateHeterogeneity(params)
```

---

**Description**

Plots the actual and expected number of customers who made a certain number of repeat transactions in the calibration period. Also returns a matrix with this comparison.
Usage

```r
bgbb.PlotFrequencyInCalibration(params, rf.matrix, censor = NULL, plotZero = TRUE, xlab = "Calibration period transactions", ylab = "Customers", title = "Frequency of Repeat Transactions")
```

Arguments

- `params`: BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
- `rf.matrix`: recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), number of transaction opportunities in the calibration period ("n.cal"), and the number of customers with this combination of recency, frequency and transaction opportunities in the calibration period ("custs"). Note that recency must be the time between the start of the calibration period and the customer's last transaction, not the time between the customer's last transaction and the end of the calibration period.
- `censor`: optional. Any calibration period frequency at this number, or above it, will be binned together. If the censor number is greater than the maximum recency in the recency-frequency matrix, the maximum recency will be used as the censor number.
- `plotZero`: If FALSE, the histogram will exclude the zero bin.
- `xlab`: descriptive label for the x axis.
- `ylab`: descriptive label for the y axis.
- `title`: title placed on the top-center of the plot.

Value

Calibration period repeat transaction frequency comparison matrix, actual vs. expected.

References


Examples

```r
data(donationsSummary)
rf.matrix <- donationsSummary$rf.matrix  # donationsSummary$rf.matrix already has appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)
```
Description

Plots the actual and expected number of customers who made a certain number of transactions in the holdout period, binned according to holdout period frequencies. Also returns a matrix with this comparison and the number of customers in each bin.

Usage

bgbb.PlotFrequencyInHoldout(params, n.cal, rf.matrix.holdout, censor = NULL, plotZero = TRUE, title = "Frequency of Repeat Transactions", xlab = "Holdout period transactions", ylab = "Customers")

Arguments

params
BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

n.cal
number of transaction opportunities in the calibration period.

rf.matrix.holdout
holdout period recency-frequency matrix. It must contain columns for frequency in the holdout period ("x.star"), the number of transaction opportunities in the holdout period ("n.star"), and the number of customers with each frequency ("custs").

censor
optional. Any calibration period frequency at this number, or above it, will be binned together. If the censor number is greater than the maximum recency in the recency-frequency matrix, the maximum recency will be used as the censor number.

plotZero
If FALSE, the histogram will exclude the zero bin.

title
title placed on the top-center of the plot.

xlab
descriptive label for the x axis.

ylab
descriptive label for the y axis.

Value

Holdout period repeat transaction frequency comparison matrix (actual vs. expected).
References


Examples

data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix
rf.matrix.holdout <- donationsSummary$rf.matrix.holdout
# donationsSummary$rf.matrix and donationsSummary$rf.matrix.holdout already
# have appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)

# number of periods in the calibration period
n.cal = max(rf.matrix[,"n.cal"])

bgbb.PlotFrequencyInHoldout(est.params, n.cal, rf.matrix.holdout)

---

bgbb.PlotFreqVsConditionalExpectedFrequency

*BG/BB Plot Frequency vs Conditional Expected Frequency*

Description

Plots the actual and conditional expected number of transactions made by customers in the holdout period, binned according to calibration period frequencies. Also returns a matrix with this comparison and the number of customers in each bin.

Usage

`bgbb.PlotFreqVsConditionalExpectedFrequency(params, n.star, rf.matrix, x.star, 
  trunc = NULL, xlab = "Calibration period transactions", 
  ylab = "Holdout period transactions", xticklab = NULL, 
  title = "Conditional Expectation")`

Arguments

- **params**  
  *BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.*

- **n.star**  
  number of transaction opportunities in the holdout period.
rf.matrix recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), number of transaction opportunities in the calibration period ("n.cal"), and the number of customers with this combination of recency, frequency and transaction opportunities in the calibration period ("custs"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.

x.star a vector containing the number of transactions made in the holdout period by the groups of customers with the same recency and frequency in the calibration period. It must be in the same order as the rf.matrix.

trunc optional integer used to truncate the plot. In the plot, all calibration period frequencies above the truncation number will be removed. If the truncation number is greater than the maximum frequency, R will warn you and change it to the maximum frequency.

xlab descriptive label for the x axis.

ylab descriptive label for the y axis.

xticklab vector containing a label for each tick mark on the x axis.

title title placed on the top-center of the plot.

Value

Holdout period transaction frequency comparison matrix (actual vs. expected), binned by calibration period frequency.

References


Examples

data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix
# donationsSummary$rf.matrix already has appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)

# get the holdout period transactions
x.star <- donationsSummary$x.star

# number of transaction opportunities in the holdout period
n.star <- 5

# Plot holdout period transactions
bgbb.PlotRecVsConditionalExpectedFrequency

**BG/BB Plot Recency vs Conditional Expected Frequency**

**Description**

Plots the actual and conditional expected number of transactions made by customers in the holdout period, binned according to calibration period recencies. Also returns a matrix with this comparison and the number of customers in each bin.

**Usage**

```r
bgbb.PlotRecVsConditionalExpectedFrequency(est.params, n.star, rf.matrix, x.star, trunc=6)
```

**Arguments**

- **params**: BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
- **n.star**: number of transaction opportunities in the holdout period.
- **rf.matrix**: recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), number of transaction opportunities in the calibration period ("n.cal"), and the number of customers with this combination of recency, frequency and transaction opportunities in the calibration period ("custs"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.
- **x.star**: a vector containing the number of transactions made in the holdout period by the groups of customers with the same recency and frequency in the calibration period. It must be in the same order as the rf.matrix.
- **trunc**: optional integer used to truncate the plot. In the plot, all calibration period frequencies above the truncation number will be removed. If the truncation number is greater than the maximum frequency, R will warn you and change it to the maximum frequency.
- **xlab**: descriptive label for the x axis.
- **ylab**: descriptive label for the y axis.
- **xticklab**: vector containing a label for each tick mark on the x axis.
- **title**: title placed on the top-center of the plot.
### Value

Holdout period transaction frequency comparison matrix (actual vs. expected), binned according calibration period recency.

### References


### Examples

```r
data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix
# donationsSummary$rf.matrix already has appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)

# get the holdout period transactions
x.star <- donationsSummary$x.star

# number of transaction opportunities in the holdout period
n.star <- 5

# Compare holdout period transactions.
bgbb.PlotRecVsConditionalExpectedFrequency(est.params, n.star, rf.matrix, x.star, trunc=6)
```

### Description

Plots the actual and expected cumulative total repeat transactions by all customers for the calibration and holdout periods. Also returns a matrix with this comparison.

### Usage

```r
bgbb.PlotTrackingCum(params, rf.matrix, actual.cum.repeat.transactions,
                      xlab = "Time", ylab = "Cumulative Transactions", xticklab = NULL,
                      title = "Tracking Cumulative Transactions")
```
Arguments

params  BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

rf.matrix  recency-frequency matrix. It must contain columns for the number of transactions opportunities in the calibration period ("n.cal"), and the number of customers with this number of transaction opportunities in the calibration period ("custs"). Columns for frequency and recency may be in the matrix, but are not necessary for this function since it relies on bgbb.Expectation, which only requires the number of transaction opportunities.

actual.cum.repeat.transactions  vector containing the cumulative number of repeat transactions made by customers in all transaction opportunities (both calibration and holdout periods). Its unit of time should be the same as the units of the recency-frequency matrix used to estimate the model parameters.

xlab  descriptive label for the x axis.

ylab  descriptive label for the y axis.

xticklab  vector containing a label for each tick mark on the x axis.

title  title placed on the top-center of the plot.

Details

The holdout period should immediately follow the calibration period. This function assumes that all customers’ calibration periods end on the same date, rather than starting on the same date (thus customers’ birth periods are determined using max(n.cal) - n.cal rather than assuming that it is 0).

Value

Matrix containing actual and expected cumulative repeat transactions.

References


Examples

data(donationsSummary)
# donationsSummary$rf.matrix already has appropriate column names
rf.matrix <- donationsSummary$rf.matrix

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)
# get the annual repeat transactions, and transform them into
# a cumulative form
actual.inc.repeat.transactions <- donationsSummary$annual.trans
actual.cum.repeat.transactions <- cumsum(actual.inc.repeat.transactions)

# set appropriate x-axis
x.tickmarks <- c("'96","'97","'98","'99","'00","'01","'02","'03","'04","'05","'06")

# plot actual vs. expected transactions. The calibration period was 6 periods long.
bgbb.PlotTrackingCum(est.params, rf.matrix, actual.cum.repeat.transactions, xticklab=x.tickmarks)

---

**bgbb.PlotTrackingInc**  
**BG/BB Tracking Incremental Transactions Plot**

**Description**

Plots the actual and expected incremental total repeat transactions by all customers for the calibration and holdout periods. Also returns a matrix of this comparison.

**Usage**

```r
bgbb.PlotTrackingInc(params, rf.matrix, actual.inc.repeat.transactions, 
                      xlab = "Time", ylab = "Transactions", xticklab = NULL, 
                      title = "Tracking Incremental Transactions")
```

**Arguments**

- **params**  
  BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

- **rf.matrix**  
  recency-frequency matrix. It must contain columns for the number of transactions opportunities in the calibration period ("n.cal"), and the number of customers with this number of transaction opportunities in the calibration period ("custs"). Columns for frequency and recency may be in the matrix, but are not necessary for this function since it relies on `bgbb.Expectation`, which only requires the number of transaction opportunities.

- **actual.inc.repeat.transactions**  
  vector containing the incremental number of repeat transactions made by customers in all transaction opportunities (both calibration and holdout periods). Its unit of time should be the same as the units of the recency-frequency matrix used to estimate the model parameters.

- **xlab**  
  descriptive label for the x axis.

- **ylab**  
  descriptive label for the y axis.

- **xticklab**  
  vector containing a label for each tick mark on the x axis.

- **title**  
  title placed on the top-center of the plot.
Details

The holdout period should immediately follow the calibration period. This function assumes that all customers’ calibration periods end on the same date, rather than starting on the same date (thus customers’ birth periods are determined using max(n.cal) - n.cal rather than assuming that it is 0).

Value

Matrix containing actual and expected incremental repeat transactions.

References


Examples

data(donationsSummary)
# donationsSummary$rf.matrix already has appropriate column names
rf.matrix <- donationsSummary$rf.matrix

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)

# get the annual repeat transactions
actual.inc.repeat.transactions <- donationsSummary$annual.trans

# Set appropriate x-axis
x.tickmarks <- c("'96","97","98","99","00","01","02","03","04","05","06")

# Plot actual vs. expected transactions. The calibration period was 6 periods long.
bgb.b.PlotTrackingInc(est.params, rf.matrix, actual.inc.repeat.transactions, xticklab=x.tickmarks)

bgbb.PlotTransactionRateHeterogeneity

*BG/BB Plot Transaction Rate Heterogeneity*

Description

Plots and returns the estimated beta distribution of P (customers’ propensities to purchase).

Usage

bgbb.PlotTransactionRateHeterogeneity(params)
Arguments
params  BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

Details
This returns the distribution of each customer's Bernoulli parameter, which determines the level of their purchasing (using the BG/BB assumption that purchasing on the individual level can be modeled with a Bernoulli distribution).

Value
Distribution of customers' propensities to purchase.

Examples
params <- c(1.2, 0.75, 0.66, 2.78)
bgbb.PlotTransactionRateHeterogeneity(params)
params <- c(0.2, 1.5, 3.2, 6)
bgbb.PlotTransactionRateHeterogeneity(params)

bgbb.pmf  \textit{BG/BB Probability Mass Function}

Description
Probability mass function for the BG/BB.

Usage
bgbb.pmf(params, n, x)

Arguments
params  BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

n  number of transaction opportunities; may also be a vector.

x  number of transactions; may also be a vector.
Details

\[ P(X(n)=x \mid \alpha, \beta, \gamma, \delta) \]. Returns the probability that a customer makes \( x \) transactions in the first \( n \) transaction opportunities.

Parameters \( t \) and \( x \) may be vectors. The standard rules for vector operations apply - if they are not of the same length, the shorter vector will be recycled (start over at the first element) until it is as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Probability of \( X(n)=x \), conditional on model parameters.

References


See Also

* bgbb.pmf.General

Examples

```r
params <- c(1.20, 0.75, 0.66, 2.78)
# The probability that a customer made 3 transactions in the first 6 transaction opportunities.
bgbb.pmf(params, n=6, x=3)

# Vectors may also be used as arguments:
bgbb.pmf(params, n=6, x=0:6)
```

**Description**

Calculates the probability that a customer will make \( x.\text{star} \) transactions in the first \( n.\text{star} \) transaction opportunities following the calibration period.

**Usage**

```r
bgbb.pmf.General(params, n.cal, n.star, x.star)
```
Arguments

params  BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
n.cal  number of transaction opportunities in the calibration period, or a vector of calibration period transaction opportunities.
n.star  number of transaction opportunities in the holdout period, or a vector of holdout period transaction opportunities.
x.star  number of transactions in the holdout period, or a vector of transaction frequencies.

Details

P(X(n, n+n*)=x* | alpha, beta, gamma, delta). This is a more generalized version of the bgbb.pmf. Setting n.cal to 0 reduces this function to the probability mass function in its usual format - the probability that a user will make x.star transactions in the first n.star transaction opportunities.

It is impossible for a customer to make a negative number of transactions, or to make more transactions than there are transaction opportunities. This function will throw an error if such inputs are provided.

n.cal, n.star, and x.star may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Probability of X(n,n+n*)=x*, given BG/BB model parameters.

References


Examples

```r
params <- c(1.20, 0.75, 0.66, 2.78)
# Probability that a customer will make 3 transactions in the 10
# transaction opportunities following the 6 transaction opportunities
# in the calibration period, given BG/BB parameters.
bgbb.pmf.General(params, n.cal=6, n.star=10, x.star=3)

# Vectors may also be provided as input:
# Comparison between different frequencies:
bgbb.pmf.General(params, n.cal=6, n.star=10, x.star=1:10)
# Comparison between different holdout transaction opportunities:
bgbb.pmf.General(params, n.cal=6, n.star=5:15, x.star=3)
```
**bgbb.PosteriorMeanDropoutRate**

*BG/BB Posterior Mean Dropout Rate*

**Description**
Computes the mean value of the marginal posterior value of Theta, the geometric dropout process parameter.

**Usage**

\[
\text{bgbb.PosteriorMeanDropoutRate}(\text{params, x, t.x, n.cal})
\]

\[
\text{bgbb.rf.matrix.PosteriorMeanDropoutRate}(\text{params, rf.matrix})
\]

**Arguments**

- **params**: BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
- **x**: number of repeat transactions a customer made in the calibration period, or a vector of calibration period transaction frequencies.
- **t.x**: recency - the last transaction opportunity in which a customer made a transaction, or a vector of recencies.
- **n.cal**: number of transaction opportunities in the calibration period, or a vector of calibration period transaction opportunities.
- **rf.matrix**: recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), and the number of transaction opportunities in the calibration period ("n.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.

**Details**

\[
E(\Theta | \alpha, \beta, \gamma, \delta, x, t.x, n) \]

This is calculated by setting \(l=0\) and \(m=1\) in \text{bgbb.PosteriorMeanLmProductMoment}.

x, t.x, and n.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.

**Value**
The posterior mean dropout rate.
References

Examples
```r
data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix  # donationsSummary$rf.matrix already has appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)

# return the posterior mean dropout rate vector
bgbb.rf.matrix.PosteriorMeanDropoutRate(est.params, rf.matrix)
```

Description

Computes the \((l,m)\)th product moment of the joint posterior distribution of \(P\) (the Bernoulli transaction process parameter) and \(\Theta\) (the geometric dropout process parameter).

Usage

```r
bgbb.PosteriorMeanLmProductMoment(params, l, m, x, t.x, n.cal)
```

Arguments

- **params** BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.
- **l** moment degree of \(P\)
- **m** moment degree of \(\Theta\)
- **x** number of repeat transactions a customer made in the calibration period, or a vector of calibration period transaction frequencies.
- **t.x** recency - the last transaction opportunity in which a customer made a transaction, or a vector of recencies.
- **n.cal** number of transaction opportunities in the calibration period, or a vector of calibration period transaction opportunities.
Details

\[
E((P)^l(\Theta)^m | \alpha, \beta, \gamma, \delta, x, t.x, n)
\]
x, t.x, and n.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.

Value

The expected posterior \((l,m)\)th product moment.

References


See equation 17.

---

**bgbb::PosteriorMeanTransactionRate**

*BG/BB Posterior Mean Transaction Rate*

**Description**

Computes the mean value of the marginal posterior value of \(P\), the Bernoulli transaction process parameter.

**Usage**

```r
bgbb::PosteriorMeanTransactionRate(params, x, t.x, n.cal)
```

```r
bgbb::rf.matrix::PosteriorMeanTransactionRate(params, rf.matrix)
```

**Arguments**

- `params`  
  BG/BB parameters - a vector with alpha, beta, gamma, and delta, in that order. Alpha and beta are unobserved parameters for the beta-Bernoulli transaction process. Gamma and delta are unobserved parameters for the beta-geometric dropout process.

- `x`  
  number of repeat transactions a customer made in the calibration period, or a vector of calibration period transaction frequencies.

- `t.x`  
  recency - the last transaction opportunity in which a customer made a transaction, or a vector of recencies.

- `n.cal`  
  number of transaction opportunities in the calibration period, or a vector of calibration period transaction opportunities.
rf.matrix  recency-frequency matrix. It must contain columns for frequency ("x"), recency ("t.x"), and the number of transaction opportunities in the calibration period ("n.cal"). Note that recency must be the time between the start of the calibration period and the customer's last transaction, not the time between the customer's last transaction and the end of the calibration period.

Details

\[ E(P | \alpha, \beta, \gamma, \delta, x, t.x, n) \]

This is calculated by setting \( l=1 \) and \( m=0 \) in \texttt{bgbbPosteriorMeanProductMoment}.

\( x, t.x, \) and \( n.cal \) may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.

Value

The posterior mean transaction rate.

References


Examples

```r
data(donationsSummary)

rf.matrix <- donationsSummary$rf.matrix
# donationsSummary$rf.matrix already has appropriate column names

# starting-point parameters
startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
est.params <- bgbb.EstimateParameters(rf.matrix, startingparams)

# return the posterior mean transaction rate vector
bgbb.rf.matrix.PosteriorMeanTransactionRate(est.params, rf.matrix)
```

bgnbd.compress.cbs  Compress Customer-by-Sufficient-Statistic Matrix

Description

Combines all customers with the same combination of recency, frequency and length of calibration period in the customer-by-sufficient-statistic matrix, and adds a fourth column labelled "custs" (with the number of customers belonging in each row).
bgnbd.compress.cbs

Usage

```
bgnbd.compress.cbs(cbs, rounding = 3)
```

Arguments

cbs  calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer's last transaction and the end of the calibration period.

rounding  the function tries to ensure that there are similar customers by rounding the customer-by-sufficient-statistic matrix first. This parameter determines how many decimal places are left in the data. Negative numbers are allowed; see the documentation for `round` in the base package. As of the time of writing, that documentation states: "Rounding to a negative number of digits means rounding to a power of ten, so for example `round(x, digits = -2)` rounds to the nearest hundred."

Details

This function was made for compatibility and consistency with the Pareto/NBD function, but will not provide speed gains for the BG/NBD model.

This function only takes columns "x", "t.x", and "T.cal" into account. All other columns will be added together - for example, if you have a spend column, the output’s spend column will contain the total amount spent by all customers with an identical recency, frequency, and time observed.

Value

A customer-by-sufficient-statistic matrix with an additional column "custs", which contains the number of customers with each combination of recency, frequency and length of calibration period.

Examples

```
# Create a sample customer-by-sufficient-statistic matrix:
set.seed(7)
x <- sample(1:4, 10, replace = TRUE)
t.x <- sample(1:4, 10, replace = TRUE)
T.cal <- rep(4, 10)
ave.spend <- sample(10:20, 10, replace = TRUE)
cbs <- cbind(x, t.x, T.cal, ave.spend)
cbs

# If cbs is printed, you would note that the following
# sets of rows have the same x, t.x and T.cal:
# (1, 6, 8); (3, 9)

bgnbd.compress.cbs(cbs, 0)  # No rounding necessary

# Note that all additional columns (in this case, ave.spend)
# are aggregated by sum.
```
Description

Uses BG/NBD model parameters and a customer’s past transaction behavior to return the number of transactions they are expected to make in a given time period.

Usage

\[
\text{bgnbd.\text{ConditionalExpectedTransactions}(params, T.star, x, t.x, T.cal)}
\]

Arguments

- **params**: BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
- **T.star**: length of time for which we are calculating the expected number of transactions.
- **x**: number of repeat transactions in the calibration period T.cal, or a vector of calibration period frequencies.
- **t.x**: time of most recent repeat transaction, or a vector of recencies.
- **T.cal**: length of calibration period, or a vector of calibration period lengths.

Details

\[
E[X(T.cal, T.cal + T.star) | x, t.x, r, alpha, a, b]
\]

T.star, x, t.x, and T.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Number of transactions a customer is expected to make in a time period of length t, conditional on their past behavior. If any of the input parameters has a length greater than 1, this will be a vector of expected number of transactions.

References

See Also

bgnbd.Expectation

Examples

```r
params <- c(0.243, 4.414, 0.793, 2.426)
# Number of transactions a customer is expected to make in 2 time
# intervals, given that they made 10 repeat transactions in a time period
# of 39 intervals, with the 10th repeat transaction occurring in the 35th
# interval.
bgnbd.ConditionedExpectedTransactions(params, T.star=2, x=10, t.x=35, T.cal=39)

# We can also compare expected transactions across different
# calibration period behaviors:
bgnbd.ConditionedExpectedTransactions(params, T.star=2, x=5:20, t.x=25, T.cal=39)
```

## bgnbd.EstimateParameters

**BG/NBD Parameter Estimation**

### Description

Estimates parameters for the BG/NBD model.

### Usage

```r
bgnbd.EstimateParameters(cal.cbs, par.start = c(1, 3, 1, 3),
                          max.param.value = 10000)
```

### Arguments

- `cal.cbs`: calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.

- `par.start`: initial BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.

- `max.param.value`: the upper bound on parameters.
The best-fitting parameters are determined using the `bgnbd.cbs.LL` function. The sum of the log-likelihood for each customer (for a set of parameters) is maximized in order to estimate parameters.

A set of starting parameters must be provided for this method. If no parameters are provided, (1,3,1,3) is used as a default. These values are used because they provide good convergence across data sets. It may be useful to use starting values for \( r \) and \( \alpha \) that represent your best guess of the heterogeneity in the buy and die rate of customers. It may be necessary to run the estimation from multiple starting points to ensure that it converges. To compare the log-likelihoods of different parameters, use `bgnbd.cbs.LL`.

The lower bound on the parameters to be estimated is always zero, since BG/NBD parameters cannot be negative. The upper bound can be set with the `max.param.value` parameter.

This function may take some time to run.

**Value**

Vector of estimated parameters.

**References**


**See Also**

`bgnbd.cbs.LL`

**Examples**

```r
data(cdnowSummary)
cal.cbs <- cdnowSummary$cdbs
  # cal.cbs already has column names required by method

  # starting-point parameters
  startingparams <- c(1.0, 3, 1.0, 3)
  # estimated parameters
  est.params <- bgnbd.EstimateParameters(cal.cbs, startingparams)
  # log-likelihood of estimated parameters
  bgnbd.cbs.LL(est.params, cal.cbs)
```
Description

Returns the number of repeat transactions that a randomly chosen customer (for whom we have no prior information) is expected to make in a given time period.

Usage

bgnbd.Expectation(params, t)

Arguments

params  BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.

params  length of time for which we are calculating the expected number of repeat transactions.

Details

\[ E(X(t) \mid r, \alpha, a, b) \]

Value

Number of repeat transactions a customer is expected to make in a time period of length t.

References


See Also

bgnbd.ConditionedExpectedTransactions

Examples

params <- c(0.243, 4.414, 0.793, 2.426)

# Number of repeat transactions a customer is expected to make in 2 time intervals.
# bgnbd.Expectation(params, t=2)

# We can also compare expected transactions over time:
# bgnbd.Expectation(params, t=1:10)
bgnbd.ExpectedCumulativeTransactions

BG/NBD Expected Cumulative Transactions

Description

Calculates the expected cumulative total repeat transactions by all customers for the calibration and holdout periods.

Usage

bgnbd.ExpectedCumulativeTransactions(params, T.cal, T.tot, n.periods.final)

Arguments

- **params**: BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.

- **T.cal**: a vector to represent customers’ calibration period lengths (in other words, the “T.cal” column from a customer-by-sufficient-statistic matrix).

- **T.tot**: end of holdout period. Must be a single value, not a vector.

- **n.periods.final**: number of time periods in the calibration and holdout periods. See details.

Details

The function automatically divides the total period up into n.periods.final time intervals. n.periods.final does not have to be in the same unit of time as the T.cal data. For example:

- if your T.cal data is in weeks, and you want cumulative transactions per week, n.periods.final would equal T.star.

- if your T.cal data is in weeks, and you want cumulative transactions per day, n.periods.final would equal T.star * 7.

The holdout period should immediately follow the calibration period. This function assume that all customers’ calibration periods end on the same date, rather than starting on the same date (thus customers’ birth periods are determined using max(T.cal) – T.cal rather than assuming that it is 0).

Value

Vector of expected cumulative total repeat transactions by all customers.

See Also

bgnbd.Expectation
Examples

data(cdnowSummary)

cal.cbs <- cdnowSummary$cbs
  # cal.cbs already has column names required by method

params <- c(0.243, 4.414, 0.793, 2.426)

  # Returns a vector containing cumulative repeat transactions for 273 days.
  # All parameters are in weeks; the calibration period lasted 39 weeks.
bgnbd.ExpectedCumulativeTransactions(params, T.cal=cal.cbs[,"T.cal"], T.tot=39, n.periods.final=273)

---

bgnbd.cbs

**BG/NBD Log-Likelihood**

Description

Calculates the log-likelihood of the BG/NBD model.

Usage

bgnbd.cbs.LL(params, cal.cbs)

bgnbd.LL(params, x, t.x, T.cal)

Arguments

- **params**: BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
- **cal.cbs**: calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer's last transaction, not the time between the customer's last transaction and the end of the calibration period. If your data is compressed (see `bgnbd.compress.cbs`), a fourth column labelled "custs" (number of customers with a specific combination of recency, frequency and length of calibration period) is available.
- **x**: number of repeat transactions in the calibration period T.cal, or a vector of transaction frequencies.
- **t.x**: time of most recent repeat transaction, or a vector of recencies.
- **T.cal**: length of calibration period, or a vector of calibration period lengths.
Details

x, t.x, T.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.

Value

For bgnbd.cbs.LL, the total log-likelihood of the provided data. For bgnbd.LL, a vector of log-likelihoods as long as the longest input vector (x, t.x, or T.cal).

See Also

bgnbd.EstimateParameters

Examples

data(cdnowSummary)

cal.cbs <- cdnowSummary$cbs
  # cal.cbs already has column names required by method

  # random assignment of parameters
  params <- c(0.5, 6, 1.2, 3.3)
  # returns the log-likelihood of the given parameters
  bgnbd.cbs.LL (params, cal.cbs)

  # compare the speed and results to the following:
  cal.cbs.compressed <- bgnbd.compress.cbs(cal.cbs)
  bgnbd.cbs.LL (params, cal.cbs.compressed)

  # Returns the log likelihood of the parameters for a customer who
  # made 3 transactions in a calibration period that ended at t=6,
  # with the last transaction occurring at t=4.
  bgnbd.LL(params, x=3, t.x=4, T.cal=6)

  # We can also give vectors as function parameters:
  set.seed(7)
  x <- sample(1:4, 10, replace = TRUE)
  t.x <- sample(1:4, 10, replace = TRUE)
  T.cal <- rep(4, 10)
  bgnbd.LL(params, x, t.x, T.cal)
Description

Uses BG/NBD model parameters and a customer's past transaction behavior to return the probability that they are still alive at the end of the calibration period.

Usage

bgnbd.PAlive(params, x, t.x, T.cal)

Arguments

params           BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
x               number of repeat transactions in the calibration period, or a vector of transaction frequencies.
t.x             time of most recent repeat transaction, or a vector of recencies.
T.cal           length of calibration period, or a vector of calibration period lengths.

Details

\[ P(\text{Alive} \mid X=x, t.x, T.cal, r, alpha, a, b) \]

x, t.x, and T.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Probability that the customer is still alive at the end of the calibration period. If x, t.x, and/or T.cal has a length greater than one, then this will be a vector of probabilities (containing one element matching each element of the longest input vector).

References


Examples

params <- c(0.243, 4.414, 0.793, 2.426)

bgnbd.PAlive(params, x=23, t.x=39, T.cal=39)
# P(Alive) of a customer who has the same recency and total time observed.

bgnbd.PAlive(params, x=5:20, t.x=30, T.cal=39)
# Note the "increasing frequency paradox".
# To visualize the distribution of P(Alive) across customers:

data(cdnnowSummary)
cbs = cdnowSummary$cbs
params = bgnbd.EstimateParameters(cbs, par.start = c(0.243, 4.414, 0.793, 2.426))
p.alives = bgnbd.PAlive(params, cbs[,"x"], cbs[,"t.x"], cbs[,"T.cal"])
plot(density(p.alives))
**bgnbd.PlotFrequencyInCalibration**

*BG/NBD Plot Frequency in Calibration Period*

**Description**

Plots a histogram and returns a matrix comparing the actual and expected number of customers who made a certain number of repeat transactions in the calibration period, binned according to calibration period frequencies.

**Usage**

```r
bgnbd.PlotFrequencyInCalibration(params, cal.cbs, censor, plotZero = TRUE, xlab = "Calibration period transactions", ylab = "Customers", title = "Frequency of Repeat Transactions")
```

**Arguments**

- `params`: BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
- `cal.cbs`: calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x") and total time observed ("T.cal").
- `censor`: integer used to censor the data. See details.
- `plotZero`: If FALSE, the histogram will exclude the zero bin.
- `xlab`: descriptive label for the x axis.
- `ylab`: descriptive label for the y axis.
- `title`: title placed on the top-center of the plot.

**Details**

This function requires a censor number, which cannot be higher than the highest frequency in the calibration period CBS. The output matrix will have (censor + 1) bins, starting at frequencies of 0 transactions and ending at a bin representing calibration period frequencies at or greater than the censor number. The plot may or may not include a bin for zero frequencies, depending on the `plotZero` parameter.

**Value**

Calibration period repeat transaction frequency comparison matrix (actual vs. expected).
bgnbd.PlotFreqVsConditionalExpectedFrequency

Examples

data(cdnnowSummary)

cal.cbs <- cdnowSummary$cbs
# cal.cbs already has column names required by method

# parameters estimated using bgnbd.EstimateParameters
est.params <- c(0.243, 4.414, 0.793, 2.426)
# the maximum censor number that can be used
max(cal.cbs[, “x”])

bgnbd.PlotFrequencyInCalibration(est.params, cal.cbs, censor=7)

Description

Plots the actual and conditional expected number transactions made by customers in the holdout period, binned according to calibration period frequencies. Also returns a matrix with this comparison and the number of customers in each bin.

Usage

bgnbd.PlotFreqVsConditionalExpectedFrequency(params, T.star, cal.cbs, x.star, censor, xlab = “Calibration period transactions”, ylab = “Holdout period transactions”, xticklab = NULL, title = “Conditional Expectation”)

Arguments

params BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
T.star length of then holdout period.
cal.cbs calibration period CBS (customer by sufficient statistic). It must contain columns for frequency (“x”), recency (“t.x”), and total time observed (“T.cal”). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.
x.star vector of transactions made by each customer in the holdout period.
censor integer used to censor the data. See details.
xlab descriptive label for the x axis.
ylab descriptive label for the y axis.
xticklab vector containing a label for each tick mark on the x axis.
title title placed on the top-center of the plot.
Details

This function requires a censor number, which cannot be higher than the highest frequency in the calibration period CBS. The output matrix will have (censor + 1) bins, starting at frequencies of 0 transactions and ending at a bin representing calibration period frequencies at or greater than the censor number.

Value

Holdout period transaction frequency comparison matrix (actual vs. expected).

Examples

data(cdnnowSummary)

    cal.cbs <- cdnowSummary$cbs
    # cal.cbs already has column names required by method

    # number of transactions by each customer in the 39 weeks
    # following the calibration period
    x.star <- cal.cbs[,"x.star"]

    # parameters estimated using bgnbd.EstimateParameters
    est.params <- c(0.243, 4.414, 0.793, 2.426)
    # the maximum censor number that can be used
    max(cal.cbs[,"x"])

    # plot conditional expected holdout period frequencies,
    # binned according to calibration period frequencies
    bgnbd.PlotFreqVsConditionalExpectedFrequency(est.params, T.star=39, cal.cbs, x.star, censor=7)

Description

Plots the actual and conditional expected number of transactions made by customers in the holdout period, binned according to calibration period recencies. Also returns a matrix with this comparison and the number of customers in each bin.

Usage

bgnbd.PlotRecVsConditionalExpectedFrequency(params, cal.cbs, T.star, x.star, xlab = "Calibration period recency", ylab = "Holdout period transactions", xticklab = NULL, title = "Actual vs. Conditional Expected Transactions by Recency")
Arguments

params | BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.

cal.cbs | calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.

T.star | length of then holdout period.

x.star | vector of transactions made by each customer in the holdout period.

xlab | descriptive label for the x axis.

ylab | descriptive label for the y axis.

xticklab | vector containing a label for each tick mark on the x axis.

title | title placed on the top-center of the plot.

Details

This function does bin customers exactly according to recency; it bins customers according to integer units of the time period of cal.cbs. Therefore, if you are using weeks in your data, customers will be binned as follows: customers with recencies between the start of the calibration period (inclusive) and the end of week one (exclusive); customers with recencies between the end of week one (inclusive) and the end of week two (exclusive); etc.

The matrix and plot will contain the actual number of transactions made by each bin in the holdout period, as well as the expected number of transactions made by that bin in the holdout period, conditional on that bin’s behavior during the calibration period.

Value

Matrix comparing actual and conditional expected transactions in the holdout period.

Examples

data(cdnnowSummary)

cal.cbs <- cdnowSummary$CBS
  # cal.cbs already has column names required by method

  # number of transactions by each customer in the 39 weeks following the calibration period
  x.star <- cal.cbs[,]"x.star"

  # parameters estimated using bgnbd.EstimateParameters
  est.params <- c(0.243, 4.414, 0.793, 2.426)

  # plot conditional expected holdout period transactions, binned according to calibration period recencies
  bgnbd.PlotRecVsConditionalExpectedFrequency(est.params, cal.cbs, T.star=39, x.star)
bgnbd.PlotTrackingCum  

**BG/NBD Tracking Cumulative Transactions Plot**

**Description**

Plots the actual and expected cumulative total repeat transactions by all customers for the calibration and holdout periods, and returns this comparison in a matrix.

**Usage**

```r
bgnbd.PlotTrackingCum(params, T.cal, T.tot, actual.cu.tracking.data,
                      xlab = "Week", ylab = "Cumulative Transactions", xticklab = NULL,
                      title = "Tracking Cumulative Transactions")
```

**Arguments**

- `params`: BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
- `T.cal`: a vector to represent customers' calibration period lengths (in other words, the "T.cal" column from a customer-by-sufficient-statistic matrix).
- `T.tot`: end of holdout period. Must be a single value, not a vector.
- `actual.cu.tracking.data`: vector containing the cumulative number of repeat transactions made by customers for each period in the total time period (both calibration and holdout periods). See details.
- `xlab`: descriptive label for the x axis.
- `ylab`: descriptive label for the y axis.
- `xticklab`: vector containing a label for each tick mark on the x axis.
- `title`: title placed on the top-center of the plot.

**Details**

- `actual.cu.tracking.data` does not have to be in the same unit of time as the `T.cal` data. `T.tot` will automatically be divided into periods to match the length of `actual.cu.tracking.data`. See `bgnbd.ExpectedCumulativeTransactions`.

  The holdout period should immediately follow the calibration period. This function assume that all customers' calibration periods end on the same date, rather than starting on the same date (thus customers' birth periods are determined using `max(T.cal) - T.cal` rather than assuming that it is 0).

**Value**

Matrix containing actual and expected cumulative repeat transactions.
Examples

```r
data(cdnnowSummary)

cal.cbs <- cdnowSummary$cbs
# cal.cbs already has column names required by method

# Cumulative repeat transactions made by all customers across calibration # and holdout periods
cu.tracking <- cdnowSummary$cu.tracking

# parameters estimated using bgnbd.EstimateParameters
est.params <- c(0.243, 4.414, 0.793, 2.426)

# All parameters are in weeks; the calibration period lasted 39 # weeks and the holdout period another 39.
bgnbd.PlotTrackingCum(est.params, T.cal=cal.cbs[,"T.cal"], T.tot=78, cu.tracking)
```

**bgnbd.PlotTrackingInc**  
BG/NBD Tracking Incremental Transactions Comparison

**Description**

Plots the actual and expected incremental total repeat transactions by all customers for the calibration and holdout periods, and returns this comparison in a matrix.

**Usage**

```r
bgnbd.PlotTrackingInc(params, T.cal, T.tot, actual.inc.tracking.data, 
                      xlab = "Week", ylab = "Transactions", xticklab = NULL, 
                      title = "Tracking Weekly Transactions")
```

**Arguments**

- `params`: BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
- `T.cal`: a vector to represent customers’ calibration period lengths (in other words, the "T.cal" column from a customer-by-sufficient-statistic matrix).
- `T.tot`: end of holdout period. Must be a single value, not a vector.
- `actual.inc.tracking.data`: vector containing the incremental number of repeat transactions made by customers for each period in the total time period (both calibration and holdout periods). See details.
- `xlab`: descriptive label for the x axis.
- `ylab`: descriptive label for the y axis.
- `xticklab`: vector containing a label for each tick mark on the x axis.
- `title`: title placed on the top-center of the plot.
**Details**

actual.inc.tracking.data does not have to be in the same unit of time as the T.cal data. T.tot will automatically be divided into periods to match the length of actual.inc.tracking.data. See bgnbd.ExpectedCumulativeTransactions.

The holdout period should immediately follow the calibration period. This function assume that all customers’ calibration periods end on the same date, rather than starting on the same date (thus customers’ birth periods are determined using max(T.cal) - T.cal rather than assuming that it is 0).

**Value**

Matrix containing actual and expected incremental repeat transactions.

**Examples**

data(cdnowSummary)
cal.cbs <- cdnowSummary$cbs
# cal.cbs already has column names required by method

# Cumulative repeat transactions made by all customers across calibration # and holdout periods
cu.tracking <- cdnowSummary$cu.tracking
# make the tracking data incremental
inc.tracking <- dc.CumulativeToIncremental(cu.tracking)

# parameters estimated using bgnbd.EstimateParameters
est.params <- c(0.243, 4.414, 0.793, 2.426)

# All parameters are in weeks; the calibration period lasted 39
# weeks and the holdout period another 39.
bgnbd.PlotTrackingInc(est.params, T.cal=cal.cbs[,"T.cal"], T.tot=78, inc.tracking)

---

**Description**

Plots and returns the estimated gamma distribution of lambda (customers’ propensities to purchase).

**Usage**

bgnbd.PlotTransactionRateHeterogeneity(params, lim = NULL)
Arguments

params  BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.

Details

This returns the distribution of each customer's Poisson parameter, which determines the level of their purchasing.

Value

Distribution of customers' propensities to purchase.

Examples

```r
params <- c(0.243, 4.414, 0.793, 2.426)
bgnbd.PlotTransactionRateHeterogeneity(params)
params <- c(0.53, 4.414, 0.793, 2.426)
bgnbd.PlotTransactionRateHeterogeneity(params)
```

Description

Probability mass function for the BG/NBD.

Usage

```r
bgnbd.pmf(params, t, x)
```

Arguments

params  BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.

t  length end of time period for which probability is being computed. May also be a vector.

x  number of repeat transactions by a random customer in the period defined by t. May also be a vector.
Details

\[ P(X(t)=x \mid r, \alpha, a, b) \]. Returns the probability that a customer makes \( x \) repeat transactions in the time interval \((0, t]\).

Parameters \( t \) and \( x \) may be vectors. The standard rules for vector operations apply - if they are not of the same length, the shorter vector will be recycled (start over at the first element) until it is as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Probability of \( X(t)=x \) conditional on model parameters. If \( t \) and/or \( x \) has a length greater than one, a vector of probabilities will be returned.

References


Examples

```r
params <- c(0.243, 4.414, 0.793, 2.426)
# probability that a customer will make 10 repeat transactions in the
# time interval (0,2]
bgnbd.pmf(params, t=2, x=10)
# probability that a customer will make no repeat transactions in the
# time interval (0,39]
bgnbd.pmf(params, t=39, x=0)

# Vectors may also be used as arguments:
bgnbd.pmf(params, t=30, x=1:20)
```

---

**bgnbd.pmf.General**

**Generalized BG/NBD Probability Mass Function**

**Description**

Generalized probability mass function for the BG/NBD.

**Usage**

\[ bgnbd.pmf.General(params, t.start, t.end, x) \]
Arguments

params  BG/NBD parameters - a vector with r, alpha, a, and b, in that order. r and alpha are unobserved parameters for the NBD transaction process. a and b are unobserved parameters for the Beta geometric dropout process.
t.start  start of time period for which probability is being calculated. It can also be a vector of values.
t.end  end of time period for which probability is being calculated. It can also be a vector of values.
x  number of repeat transactions by a random customer in the period defined by (t.start, t.end]. It can also be a vector of values.

Details

\[ P(X(t.start, t.end)=x | r, alpha, a, b). \] Returns the probability that a customer makes x repeat transactions in the time interval (t.start, t.end].

It is impossible for a customer to make a negative number of repeat transactions. This function will return an error if it is given negative times or a negative number of repeat transactions. This function will also return an error if t.end is less than t.start.

t.start, t.end, and x may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Probability of x transaction occurring between t.start and t.end conditional on model parameters. If t.start, t.end, and/or x has a length greater than one, a vector of probabilities will be returned.

References


Examples

```r
params <- c(0.243, 4.414, 0.793, 2.426)
# probability that a customer will make 10 repeat transactions in the
# time interval [1,2]
bgnbd.pmf General(params, t.start=1, t.end=2, x=10)
# probability that a customer will make no repeat transactions in the
# time interval [39,78]
bgnbd.pmf General(params, t.start=39, t.end=78, x=0)
```


**cdnowElog**  
*CDNOW event log data*

**Description**

Data representing the purchasing behavior of 2357 CDNOW customers between January 1997 and June 1998, in event log format.

**Format**

A comma-delimited file representing an event log with 6919 entries. It has 5 columns: The customer’s ID in the master dataset, the customer’s ID in this dataset (which represents 1/10th of the master dataset), the date of the transaction in the format “%Y%m%d” (e.g. 19970225), the number of CDs purchased, and the dollar value of the transaction.

**Details**

The customers in this data represent 1/10th of the cohort of customers who made their first transactions with CDNOW in the first quarter of 1997. CDNOW was an online retailer, selling music and related products on the web since 1994.

**Source**

Can be found online at [www.brucehardie.com/datasets](http://www.brucehardie.com/datasets).

---

**cdnowSummary**  
*CDNOW repeat transaction data summary*

**Description**

Data representing the purchasing behavior of 2357 CDNOW customers between January 1997 and June 1998, summarized as a customer-by-time matrix and a vector of cumulative weekly transactions.

**Usage**

```r
data(cdnowSummary)
```

**Format**

$\texttt{cbs}$  A customer-by-time matrix with four columns: frequency (“x”), recency (“t.x”), length of observation in the calibration period (“T.cal”), and number of transactions in the holdout period (“x.star”). Each row represents a customer.

$\texttt{cu.tracking}$  A vector containing cumulative transactions for every week in both the calibration and estimating periods (78 weeks total). This vector contains the sum of transactions across all customers.
$est.params$ A vector containing estimated values for the four Pareto/NBD parameters: r, alpha, s, and beta, in that order. This estimation was made using pnbd.EstimateParameters, and is included here to avoid having to run the relatively time-consuming parameter estimation function in examples.

$m.x$ A vector containing the average value of each customer's repeat transactions. Used in examples for spend functions.

**Details**

The customers in this data represent 1/10th of the cohort of customers who made their first transactions with CDNOW in the first quarter of 1997. CDNOW was an online retailer, selling music and related products on the web since 1994.

**Source**

The data was put together using data conversion functions included in this package. The original event log is included (see cdnowElod).

dc.BuildCBSFromCBTAndDates

*Build CBS matrix from CBT matrix*

**Description**

Given a customer-by-time matrix, yields the resulting customer-by-sufficient-statistic matrix.

**Usage**

dc.BuildCBSFromCBTAndDates(cbt, dates, per, cbt.is.during.cal.period = TRUE)

**Arguments**

cbt
customer-by-time matrix. This is a matrix consisting of a row per customer and a column per time period. It should contain numeric information about a customer's transactions in every time period - either the number of transactions in that time period (frequency), a 1 to indicate that at least 1 transaction occurred (reach), or the average/total amount spent in that time period.

dates
if cbt.is.during.cal.period is TRUE, then dates is a data frame with three columns: 1. the dates when customers made their first purchases 2. the dates when customers made their last purchases 3. the date of the end of the calibration period. if cbt.is.during.cal.period is FALSE, then dates is a vector with two elements: 1. the date of the beginning of the holdout period 2. the date of the end of the holdout period.

per
interval of time for customer-by-sufficient-statistic matrix. May be "day", "week", "month", "quarter", or "year".

cbt.is.during.cal.period
if TRUE, indicates the customer-by-time matrix is from the calibration period. If FALSE, indicates the customer-by-time matrix is from the holdout period.
Details

The customer-by-sufficient statistic matrix will contain the sum of the statistic included in the customer-by-time matrix (see the cbt parameter), the customer’s last transaction date, and the total time period for which the customer was observed.

Value

Customer-by-sufficient-statistic matrix, with three columns: frequency("x"), recency("t.x") and total time observed("T.cal"). See details. Frequency is total transactions, not repeat transactions.

Examples

eog <- dc.ReadLines(system.file("data/chnowelog.csv", package="BTYD"), 2, 3, 5)
eog[, "date"] <- as.Date(eog[, "date"], "%Y-%m-%d")

# Transaction-flow models are about interpurchase times. Since we
# only know purchase times to the day, we merge all transaction on
# the same day. This example uses dc.MergeTransactionsOnSameDate
# to illustrate this; however, we could have simply used dc.CreateReachCBT
# instead of dc.CreateFreqCBT to obtain the same result.
merged.eog <- dc.MergeTransactionsOnSameDate(eog)
cutoff.date <- as.Date("1997-09-30")

freq.cbt <- dc.CreateFreqCBT(merged.eog)
cal.freq.cbt <- freq.cbt[, as.Date(colnames(freq.cbt)) <= cutoff.date]
holdout.freq.cbt <- freq.cbt[, as.Date(colnames(freq.cbt)) > cutoff.date]

cal.start.dates.indices <- dc.GetFirstPurchasePeriodsFromCBT(cal.freq.cbt)
cal.end.dates.indices <- dc.GetLastPurchasePeriodsFromCBT(cal.freq.cbt)
T.cal.total <- rep(cutoff.date, nrow(cal.freq.cbt))
cal.dates <- data.frame(cal.start.dates, cal.end.dates, T.cal.total)

# Create calibration period customer-by-sufficient-statistic data frame,
# using weeks as the unit of time.
cal.cbs <- dc.BuildCBSFromCBTAndDates(cal.freq.cbt, cal.dates,
per="week", cbt.is.during.cal.period=TRUE)

# Force the calibration period customer-by-sufficient-statistic to only contain
# repeat transactions (required by BG/BB and Pareto/NBD models)
cal.cbs[, "x"] <- cal.cbs[, "x"] - 1

holdout.start <- cutoff.date + 1
holdout.end <- as.Date(colnames(holdout.freq.cbt)[ncol(holdout.freq.cbt)])
holdout.dates <- c(holdout.start, holdout.end)

# Create holdout period customer-by-sufficient-statistic data frame, using weeks
# as the unit of time.
holdout.cbs <- dc.BuildCBSFromCBTAndDates(holdout.freq.cbt, holdout.dates,
per="week", cbt.is.during.cal.period=FALSE)
Build Customer-by-Time Matrix from Event Log

**Description**

Creates a customer-by-time matrix from an event log.

**Usage**

```r
dc.BuildCBTFromElog(elog, statistic = "freq")
```

**Arguments**

- `elog`: event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction. For the total spend and average spend matrices, the event log must have a "sales" column. If the dates are not formatted to be in the order year-month-day, the columns of the customer-by-time matrix may not be ordered chronologically if the "date" column does not consist of date objects (R will order them alphabetically). This will cause problems with other functions, so it is better to convert the date column to date objects before running this function.

- `statistic`: either "freq", "reach", "total.spend", or "average.spend". This determines what type of customer-by-time matrix is returned.

**Value**

Customer-by-time matrix.

Create Frequency Customer-by-Time Matrix

**Description**

Creates a customer-by-time matrix with total number of transactions per time period.

**Usage**

```r
dc.CreateFreqCBT(elog)
```
Arguments

elog event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction. If the dates are not formatted to be in the order year-month-day, the columns of the customer-by-time matrix may not be ordered chronologically if the "date" column does not consist of date objects (R will order them alphabetically). This will cause problems with other functions, so it is better to convert the date column to date objects before running this function.

Value

Frequency customer-by-time matrix.

Examples

# Create event log from file "cdnowEl og.csv", which has
# customer IDs in the second column, dates in the third column, and
# sales numbers in the fifth column.
elog <- dc.Readlines(system.file("data/cdnowEl og.csv", package="BTYD"),2,3,5)

# Given that the dates are in the order year-month-day,
# it is not strictly necessary to convert the date column
# to date formats. However, it is good practice:
elog[,"date"] <- as.Date(elog[,"date"], "%Y%m%d")

freq.cbt <- dc.CreateFreqCBT(elog)

---

dc.CreateReachCBT Create Reach Customer-by-Time Matrix

Description

Creates a customer-by-time matrix with 1's in periods that a customer made a transaction and 0's otherwise.

Usage

dc.CreateReachCBT(elog)

Arguments

elog event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction. If the dates are not formatted to be in the order year-month-day, the columns of the customer-by-time matrix may not be ordered chronologically if the "date" column does not consist of date objects (R will order them alphabetically). This will cause problems with other functions, so it is better to convert the date column to date objects before running this function.
dc.CreateSpendCBT

Value

Reach customer-by-time matrix.

Examples

# Create event log from file "cdnowElog.csv", which has
# customer IDs in the second column, dates in the third column, and
# sales numbers in the fifth column.
elog <- dc.ReadLines(system.file("data/cdnowElog.csv", package="BTYD"), 2, 3, 5)

# Given that the dates are in the order year-month-day,
# it is not strictly necessary to convert the date column
# to date formats. However, it is good practice:
elog[, "date"] <- as.Date(elog[, "date"], "%Y%m%d")

reach.cbt <- dc.CreateReachCBT(elog)

dc.CreateSpendCBT  Create Spend Customer-by-Time Matrix

Description

Creates a customer-by-time matrix with spend per time period.

Usage

dc.CreateSpendCBT(elog, is.avg.spend = FALSE)

Arguments

elog  event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction. If the dates are not formatted to be in the order year-month-day, the columns of the customer-by-time matrix may not be ordered chronologically if the "date" column does not consist of date objects (R will order them alphabetically). This will cause problems with other functions, so it is better to convert the date column to date objects before running this function.

is.avg.spend  if TRUE, return average spend customer-by-time matrix; else, return total spend customer-by-time matrix.

Value

Spend customer-by-time matrix.
Examples

# Create event log from file "cdnowElog.csv", which has
# customer IDs in the second column, dates in the third column, and
# sales numbers in the fifth column.
elog <- dc.ReadLines(system.file("data/cdnowElog.csv", package="BTYD"), 2, 3, 5);

# Given that the dates are in the order year-month-day,
# it is not strictly necessary to convert the date column
# to date formats. However, it is good practice:
elog[,]$date <- as.Date(elog[,]$date, "%Y%m%d")

spend.cbt <- dc.CreateSpendCBT(elog)

---

dc.CumulativeToIncremental

*Cumulative to Incremental*

**Description**

Converts a vector of cumulative transactions to a vector of incremental transactions.

**Usage**

dc.CumulativeToIncremental(cu)

**Arguments**

cu vector containing cumulative transactions over time.

**Value**

Vector of incremental transactions.

---

dc.DissipateElog

*Dissipate Event Log*

**Description**

Filters an event log, keeping a fraction of the original event log.

**Usage**

dc.DissipateElog(elog, dissipate.factor)
Arguments

elog event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction.

dissipate.factor integer indicating how much of the dataset to eliminate. It must be greater than 1 for the function to work. (dissipate.factor-1)/(dissipate.factor) events will be removed from the event log. For example, if 2 is provided, 1/2 of the event log is eliminated, and if 10 is provided, 9/10 of the event log is eliminated.

Value

Reduced event log.

dc.ElogToCbsCbt

Convernt Event Log to CBS and CBT Matrices

Description

Uses an event log to return calibration period CBT and CBS, holdout period CBT and CBS, and summary data for each customer (including times of first and last transactions).

Usage

dc.ElogToCbsCbt(elog, per = "week", T.cal = max(elog$date), T.tot = max(elog$date), merge.same.date = TRUE, cohort.birth.per = T.cal, dissipate.factor = 1, statistic = "freq")

Arguments

elog event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction. The "date" column must contain date objects, not character strings or factors.

per interval of time for customer-by-sufficient-statistic matrix. May be "day", "week", "month", "quarter", or "year".

T.cal R date object indicating when the calibration period ends.

T.tot R date object indicating when holdout period ends.

merge.same.date If TRUE, transactions from the same period count as a single transaction instead of counting as multiple transactions.

cohort.birth.per Time interval used to filter the event log. Can be specified as a Date object or a vector of two Dates. If one date object is used, the birth period is from the minimum date in the dataset through the given date. If two dates are given, the birth period is set between (inclusive) the two dates.
dc.FilterCustByBirth

**dissipate.factor**

integer indicating how much of the dataset to eliminate. If left as 1, none of the dataset is eliminated. \((\text{dissipate.factor}-1)/\text{dissipate.factor}\) events will be removed from the event log. For example, if 2 is provided, 1/2 of the event log is eliminated, and if 10 is provided, 9/10 of the event log is eliminated.

**statistic**

Determines type of CBT returned: can be: "reach", "freq", "total.spend", or "average.spend." (note: spend requires $sales column in elog)

**Details**

This function automatically removes customers’ first transactions, meaning that the output matrices will only contain repeat transaction information.

**Value**

A list of items:

- `$cal` list with CBS and CBT from the calibration period
- `$holdout` list with CBS and CBT from holdout period
- `$cust.data` data frame with each customer’s first and last transaction details

**Examples**

```R
# Create event log from file "cdnowElog.csv", which has
# customer IDs in the second column, dates in the third column, and
# sales numbers in the fifth column.
elog <- dc.ReadLines(system.file("data/cdnowElog.csv", package="BYD"),2,3,5)
elog[,"date"] <- as.Date(elog[,"date"], "%Y%m%d")
data <- dc.ElogToCbsCbt(elog, per="week", T.cal=as.Date("1997-09-30"))
```

---

**dc.FilterCustByBirth**  
*Filter Customer by Birth*

**Description**

Filters an event log, keeping all transactions made by customers who made their first transactions in the given time interval.

**Usage**

`dc.FilterCustByBirth(elog, cohort.birth.per)`
dc.GetFirstPurchasePeriodsFromCBT

Get First Purchase Periods from Customer-by-Time Matrix

Description

Uses a customer-by-time matrix to return a vector containing the periods in which customers made their first purchase.

Usage

dc.GetFirstPurchasePeriodsFromCBT(cbt)

Arguments

cbt
customer-by-time matrix. This is a matrix consisting of a row per customer and a column per time period. It should contain numeric information about a customer’s transactions in every time period - either the number of transactions in that time period (frequency), a 1 to indicate that at least 1 transaction occurred (reach), or the average/total amount spent in that time period.
dc.MakeRFmatrixCal

Value

a vector containing the indices of periods in which customers made their first transactions. To convert to actual dates (if your customer-by-time matrix has dates as column names), use colnames(cbt)[RESULT]

dc.GetLastPurchasePeriodsFromCBT

Get Last Purchase Periods from Customer-by-Time Matrix

Description

Uses a customer-by-time matrix to return a vector containing the periods in which customers made their last purchase.

Usage

dc.GetLastPurchasePeriodsFromCBT(cbt)

Arguments

cbt customer-by-time matrix. This is a matrix consisting of a row per customer and a column per time period. It should contain numeric information about a customer’s transactions in every time period - either the number of transactions in that time period (frequency), a 1 to indicate that at least 1 transaction occurred (reach), or the average/total amount spent in that time period.

Value

a vector containing the indices of periods in which customers made their last transactions. To convert to actual dates (if your customer-by-time matrix has dates as column names), use colnames(cbt)[RESULT]

dc.MakeRFmatrixCal Make Calibration Period Recency-Frequency Matrix

Description

Make a calibration period recency-frequency matrix.

Usage

dc.MakeRFmatrixCal(frequencies, periods.of.final.purchases, num.of.purchase.periods, holdout.frequencies = NULL)
Arguments

frequencies vector which indicates the number of repeat transactions made by customers in the calibration period.

periods.of.final.purchases
  a vector indicating in which period customers made their final purchases.

num.of.purchase.periods
  the number of transaction opportunities in the calibration period.

holdout.frequencies
  an optional vector indicating the number of transactions made by customers in the holdout period.

Value

A matrix with all possible frequency-recency combinations, and the number of customers with each combination. It contains columns for frequency ("x"), recency ("t.x"), number of transaction opportunities in the calibration period ("n.cal"), number of customers with this combination of recency, frequency, and number of periods observed ("custs"), and optionally, number of transactions in the holdout period ("x.star").

Examples

elog <- dc.ReadLines(system.file("data/discreteSimElog.csv", package="BTYD"),1,2)
elog[,"date"] <- as.Date(elog[,"date"])
cutoff.date <- as.Date("1977-01-01")
cbt <- dc.CreateReachCBT(elog)
cal.cbt <- cbt[,as.Date(colnames(cbt)) <= cutoff.date]
holdout.cbt <- cbt[,as.Date(colnames(cbt)) > cutoff.date]

cal.start.dates.indices <- dc.GetFirstPurchasePeriodsFromCBT(cal.cbt)
cal.start.dates <- as.Date(colnames(cal.cbt)[cal.start.dates.indices])
cal.end.dates.indices <- dc.GetLastPurchasePeriodsFromCBT(cal.cbt)
cal.end.dates <- as.Date(colnames(cal.cbt)[cal.end.dates.indices])
T.cal.total <- rep(cutoff.date, nrow(cal.cbt))
cal.dates <- data.frame(cal.start.dates, cal.end.dates, T.cal.total)

# Create calibration period customer-by-sufficient-statistic data frame,
# using years as the unit of time.
cal.cbs <- dc.BuildCBSFromCBTAndDates(cal.cbt, cal.dates, per="year",
cbt.is.during.cal.period=TRUE)

holdout.start <- as.Date(colnames(holdout.cbt)[1])
holdout.end <- as.Date(tail(colnames(holdout.cbt),n=1))
# The (-1) below is to remove the effect of the birth period - we are only
# interested in repeat transactions in the calibration period.
frequencies <- (cal.cbs[,"x"] - 1)
periods.of.final.purchases <- cal.cbs[,"t.x"]
num.of.purchase.periods <- ncol(cal.cbt) - 1

# Create a calibration period recency-frequency matrix
dc.MakeRFmatrixSkeleton

\[ \text{cal.rf.matrix} \leftarrow \text{dc.MakeRFmatrixCal}(\text{frequencies, periods.of.final.purchases, num.of.purchase.periods}) \]

dc.MakeRFmatrixHoldout

*Make Holdout Period Recency-Frequency Matrix*

**Description**

Creates a recency-frequency matrix for the holdout period.

**Usage**

\[ \text{dc.MakeRFmatrixHoldout}(\text{holdout.cbt}) \]

**Arguments**

- **holdout.cbt**: holdout period frequency customer-by-time matrix. This is a matrix consisting of a row per customer and a column per time period. It should contain the number of transactions each customer made per time period.

**Value**

recency-frequency matrix for the holdout period, with three columns: frequency ("x.star"), recency ("t.x.star"), number of transaction opportunities in the holdout period ("n.star"), and the number of customers with each frequency-recency combination ("custs").

dc.MakeRFmatrixSkeleton

*Make Recency-Frequency Matrix Skeleton*

**Description**

Creates a matrix with all possible recency and frequency combinations.

**Usage**

\[ \text{dc.MakeRFmatrixSkeleton}(\text{n.periods}) \]

**Arguments**

- **n.periods**: number of transaction opportunities in the calibration period.

**Details**

Makes the structure in which to input data for recency-frequency matrices.
Value

Matrix with two columns: frequency ("x") and recency ("t.x"). All possible recency-frequency combinations in the calibration period are represented.

Description

Takes two CBT or CBS matrices and ensures that the second one has the same row names as the first.

Usage

dc.MergeCustomers(data.correct, data.to.correct)

Arguments

data.correct  CBT or CBS with the correct customer IDs as row names. Usually from the calibration period.
data.to.correct CBT or CBS which needs to be fixed (customer IDs inserted). Usually from the holdout period.

Details

Care should be taken in using this function. It inserts zero values in all rows that were not in the original holdout period data. This behavior does not cause a problem if using CBT matrices, but will cause a problem if using CBS matrices (for example, the output will report all customers with a holdout period length of zero). However, this particular issue is easily fixed (see examples) and should not cause problems.

A work-around to avoid using this function is presented in the example for dc.BuildCBSFromCBTAndDates - build the full CBT and only use the columns applying to each particular time period to construct separate CBTs, and from them, CBSs. That is a much cleaner and less error-prone method; however, on occasion the data will not be available in event log format and you may not be able to construct a CBT for both time periods together.

Value

Updated holdout period CBT or CBS.
Examples

```r
eol <- dc.ReadLines(system.file("data/cdnwElog.csv", package="BTOY"), 2, 3, 5)
eol[, "date"] <- as.Date(eol[, "date"], "%Y-%m-%d")
cutoff.date <- as.Date("1997-09-30")
cal.eol <- eol[which(eol[, "date"] <= cutoff.date),]
holdout.eol <- eol[which(eol[, "date"] > cutoff.date),]

# Create calibration period CBT from cal.eol
cal.reach.cbt <- dc.CreateReachCBT(cal.eol)
# Create holdout period CBT from holdout.eol
holdout.reach.cbt <- dc.CreateReachCBT(holdout.eol)

# Note the difference:
nrow(cal.reach.cbt)       # 2357 customers
nrow(holdout.reach.cbt)   # 684 customers

# Create a "fixed" holdout period CBT, with the same number
# of customers in the same order as the calibration period CBT
fixed.holdout.reach.cbt <- dc.MergeCustomers(cal.reach.cbt, holdout.reach.cbt)
nrow(fixed.holdout.reach.cbt) # 2357 customers

# You can verify that the above is correct by turning these into a CBS
# (see \code{\link{dc.BuildCBSFromCBTAndDates}) and using
# \code{\link{pnbd.PlotFreqVsConditionalExpectedFrequency}}, for example

# Alternatively, we can fix the CBS, instead of the CBS:
cal.start.dates.indices <- dc.GetFirstPurchasePeriodsFromCBT(cal.reach.cbt)
(cal.start.dates <- as.Date(colnames(cal.reach.cbt)[cal.start.dates.indices]))
cal.end.dates.indices <- dc.GetLastPurchasePeriodsFromCBT(cal.reach.cbt)
(cal.end.dates <- as.Date(colnames(cal.reach.cbt)[cal.end.dates.indices]))
T.cal.total <- rep(cutoff.date, nrow(cal.reach.cbt))
cal.dates <- data.frame(cal.start.dates, cal.end.dates, T.cal.total)

# Create calibration period customer-by-sufficient-statistic data frame,
# using weeks as the unit of time.
cal.cbs <- dc.BuildCBSFromCBTAndDates(cal.reach.cbt, cal.dates, per="week",
cbt.is.during.cal.period=TRUE)

# Force the calibration period customer-by-sufficient-statistic to only
# contain repeat transactions (required by BG/BB and Pareto/NBD models)
cal.cbs[, "x"] <- cal.cbs[, "x"] - 1

holdout.start <- cutoff.date + 1
holdout.end <- as.Date(colnames(fixed.holdout.reach.cbt)[ncol(fixed.holdout.reach.cbt)])
holdout.dates <- c(holdout.start, holdout.end)

# Create holdout period customer-by-sufficient-statistic data frame,
# using weeks as the unit of time.
holdout.cbs <- dc.BuildCBSFromCBTAndDates(holdout.reach.cbt, holdout.dates,
per="week", cbt.is.during.cal.period=FALSE)
```
# Note the difference:
\[ \text{nrow(cal.cbs)} \quad \text{# 2357 customers} \]
\[ \text{nrow(holdout.cbs)} \quad \text{# 684 customers} \]

# Create a "fixed" holdout period CBS, with the same number
# of customers in the same order as the calibration period CBS
fixed.holdout.cbs <- dc.MergeCustomers(cal.cbs, holdout.cbs)
\[ \text{nrow(fixed.holdout.cbs)} \quad \text{# 2357 customers} \]

# Furthermore, this function will assign a zero value to all fields
# that were not in the original holdout period CBS. Since \( T.\text{star} \) is the
# same for all customers in the holdout period, we should fix that:
\[ \text{fixed.holdout.cbs}[,"T.\text{star}"] \leftarrow \text{rep(max(fixed.holdout.cbs[,"T.\text{star}"]),nrow(fixed.holdout.cbs))} \]

dc.MergeTransactionsOnSameDate

*Merge Transactions on Same Day*

**Description**

Updates an event log; any transactions made by the same customer on the same day are combined into one transaction.

**Usage**

```
dc.MergeTransactionsOnSameDate(elog)
```

**Arguments**

- `elog` event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction.

**Value**

Event log with transactions made by the same customer on the same day merged into one transaction.

dc.PlotLogLikelihoodContour

*Plot Log-Likelihood Contour*

**Description**

Makes a contour plot of a loglikelihood function that varies over two designated parameters, centered around a set of previously estimated parameters.
Usage

dc.PlotLogLikelihoodContour(loglikelihood.fcn, vary.or.fix.param, predicted.params, ..., n.divs = 3, new.dev = FALSE, num.contour.lines = 10, zoom.percent = 0.9, allow.neg.params = FALSE, param.names = c("param 1", "param 2", "param 3", "param 4"))

Arguments

loglikelihood.fcn
log-likelihood function to plot.

vary.or.fix.param
a vector of strings containing either "vary" or "fix". The parameters in the same indices as "vary" will be plotted while the other parameters will remain fixed at the estimated values. See details.

predicted.params
estimated parameters.

... all additional arguments required by the log-likelihood function. For example, bgbb.rf.matrix.LL requires rf.matrix, and pnbd.cbs.LL and bgnbd.cbs.LL require cal.cbs.

n.divs
integer representing how fine-grained the contour plot is. A higher value will produce a higher resolution plot with smoother contour lines, but will take longer to plot. n.divs also affects the boundaries of the contour plot; see details.

new.dev
if TRUE, makes a new window for each contour plot.

num.contour.lines
number of contour lines to plot in the window.

zoom.percent
determines boundaries of contour plot. See details.

allow.neg.params
if FALSE, the contour plot will not include negative values (see details). This should be set to false for the BG/BB and Pareto/NBD models.

param.names
a vector containing parameter names.

Details

The contour plot will have the first parameter labelled "vary" on the x-axis, and the second parameter labelled "vary" on the y-axis. It will extend out by (n.divs * zoom.percent) in both directions and both dimensions from the estimated parameter values. The exception is if allow.neg.params is FALSE. In this case, the contour plot will end at zero if it would have extended into negative parameter values.

The estimated parameter values will be indicated by the intersection of two red lines.

See Also

dc.PlotLogLikelihoodContours
Examples

```r
# **Examples for BG/BB model:**
data(donationsSummary)
rf.matrix <- donationsSummary$rf.matrix

# starting-point parameters
bgbb.startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
bgbb.est.params <- bgbb.EstimateParameters(rf.matrix, bgbb.startingparams)

# set up parameter names for a more descriptive result
bgbb.param.names <- c("alpha", "beta", "gamma", "delta")

# plot a log-likelihood contour of alpha and beta, the unobserved
# parameters for the beta-Bernoulli transaction process of the BG/BB.
# Note that allow.neg.params has been set to false as BG/BB parameters
# cannot be negative (not run):
dc.PlotLogLikelihoodContour(bgbb.rf.matrix$LL, c("vary", "vary", "fix", "fix"),
  bgbb.est.params, rf.matrix = rf.matrix, n.divs = 15,
  num.contour.lines = 15, zoom.percent = 0.2,
  allow.neg.params = FALSE, param.names = bgbb.param.names)

# plot a log-likelihood contour of gamma and delta, the unobserved
# parameters for the beta-geometric dropout process of the BG/BB.
# Note that allow.neg.params has been set to false as BG/BB parameters
# cannot be negative (not run):
dc.PlotLogLikelihoodContour(bgbb.rf.matrix$LL, c("fix", "fix", "vary", "vary"),
  bgbb.est.params, rf.matrix = rf.matrix, n.divs = 15,
  num.contour.lines = 15, zoom.percent = 0.2,
  allow.neg.params = FALSE, param.names = bgbb.param.names)

# **Example for Pareto/NBD model:**
data(cdnnowSummary)
cbs <- cdnowSummary$cbs

# Speed up calculations:
cbs <- pnbd.compress.cbs(cbs)

# parameters estimated using pnbd.EstimateParameters
pnbd.est.params <- cdnowSummary$est.params

# set up parameter names for a more descriptive result
pnbd.param.names <- c("r", "alpha", "s", "beta")

# plot a log-likelihood contour of r and s, the shape parameters
# of the transaction and dropout process models (respectively).
# Note that allow.neg.params has been set to false as Pareto/NBD
# parameters cannot be negative (not run):
dc.PlotLogLikelihoodContour(pnbd.cbs$LL, c("vary", "fix", "vary", "fix"),
  pnbd.est.params, cals = cbs, n.divs = 20,
  num.contour.lines = 20, zoom.percent = 0.1,
  allow.neg.params = FALSE, param.names = pnbd.param.names)
```
dc.PlotLogLikelihoodContours

Plot Log-Likelihood Contour

Description

Creates a set of contour plots, such that there is a contour plot for every pair of parameters varying.

Usage

dc.PlotLogLikelihoodContours(loglikelihood.fcn, predicted.params, ..., n.divs = 2, multiple.screens = FALSE, num.contour.lines = 10, zoom.percent = 0.9, allow.neg.params = FALSE, param.names = c("param 1", "param 2", "param 3", "param 4"))

Arguments

loglikelihood.fcn

log-likelihood function to plot.

predicted.params

estimated parameters.

... all additional arguments required by the log-likelihood function. For example, bgbb.rf.matrix.LL requires rf.matrix, and pnbd.cbs.LL and bgnbd.cbs.LL require cal.cbs.

n.divs

integer representing how fine-grained the contour plot is. A higher value will produce a higher resolution plot with smoother contour lines, but will take longer to plot. n.divs also affects the boundaries of the contour plot; see details.
multiple.screens
    if TRUE, plots each contour plot on a separate R graphics window.
num.contour.lines
    number of contour lines to plot in the window.
zoom.percent
    determines boundaries of contour plot. See details.
allow.neg.params
    if FALSE, the contour plot will not include negative values (see details). This
    should be set to false for the BG/BB and Pareto/NBD models.
param.names
    a vector containing parameter names.

Details
For each contour plot, the non-varying parameters are kept constant at the predicted values. The contour will extend out by (n.divs * zoom.percent) in both directions and both dimensions from the estimated parameter values. The exception is if allow.neg.params is FALSE. In this case, the contour plot will end at zero if it would have extended into negative parameter values. The estimated parameter values will be indicated by the intersection of two red lines.

See Also
dc.PlotLogLikelihoodContour

Examples

# **Example for BG/BB model:
data(donationsSummary)
RF.matrix <- donationsSummary$rf.matrix

# starting-point parameters
gbgb.startingparams <- c(1, 1, 0.5, 3)
# estimated parameters
gbgb.est.params <- bggb.EstimateParameters(RF.matrix, bggb.startingparams)

# set up parameter names for a more descriptive result
gbgb.param.names <- c("alpha", "beta", "gamma", "delta")

# plot-log likelihood contours (not run):
dc.PlotLogLikelihoodContours(bggb.RF.matrix.LL, bggb.est.params, RF.matrix = RF.matrix,
# n.divs = 5, num.contour.lines = 8, zoom.percent = 0.3, allow.neg.params = FALSE,
# param.names = bggb.param.names)

# **Example for Pareto/NBD model:
data(cdnowSummary)
cbs <- cdnowSummary$cbs

# Speed up calculations:
cbs <- pnbd.compress.cbs(cbs)

# parameters estimated using pnbd.EstimateParameters
pnbd.est.params <- cdnowSummary$est.params
# set up parameter names for a more descriptive result
pnbd.param.names <- c("r", "alpha", "s", "beta")

# plot log-likelihood contours (not run):
# dc.PlotLogLikelihoodContours(pnbd.cbs.LL, pnbd.est.params, cal.cbs = cbs, n.divs = 5, 
# num.contour.lines = 15, zoom.percent = 0.3, 
# allow.neg.params = FALSE, param.names = pnbd.param.names)

# **Example for BG/NBD model:
data(cdnowSummary)
cbs <- cdnowSummary$cbs

# parameters estimated using bgnbd.EstimateParameters
bgnbd.est.params <- cdnowSummary$est.params

# set up parameter names for a more descriptive result
bgnbd.param.names <- c("r", "alpha", "s", "beta")

# plot log-likelihood contours (not run):
# dc.PlotLogLikelihoodContours(bgnbd.cbs.LL, bgnbd.est.params, cal.cbs = cbs, n.divs = 5, 
# num.contour.lines = 15, zoom.percent = 0.3, 
# allow.neg.params = FALSE, param.names = bgnbd.param.names)

# dc.ReadLines

dc.ReadLines

## Description
Given a .csv file that throws errors when read in by the usual read.csv and read.table methods, loops through the file line-by-line and picks out the customer, date, and sales (optional) transaction data to return an event log.

## Usage

dc.ReadLines(csv.filename, cust.idx, date.idx, sales.idx = -1)

## Arguments

- **csv.filename**: filename of the comma-delimited file to be read. It must contain headers.
- **cust.idx**: index of the customer ID column in the comma-delimited file.
- **date.idx**: index of the date column in the comma-delimited file.
- **sales.idx**: index of the sales column in the comma-delimited file.

## Details
Once this function has been run, you may need to convert the date column to Date objects for the event log to work with other functions in this library. See the as.Date function in the main package for more details.
Value

Event log with columns labelled "cust" and "date". If sales data is included in the comma-delimited file, a "sales" column is included in the event log.

Examples

# Create event log from file "cdnowElog.csv", which has
# customer IDs in the second column, dates in the third column, and
# sales numbers in the fifth column.
eLog <- dc.ReadLines(system.file("data/cdnowElog.csv", package="BTYD"),2,3,5)

# convert date column to date objects, as required by some other functions
eLog$date <- as.Date(eLog$date, "Ym%d")

dc.RemoveTimeBetween  Remove Time Between

Description

This function creates a new event log, with time in the middle removed. Used, for example, in sports with off-seasons.

Usage

dc.RemoveTimeBetween(eLog, day1, day2, day3, day4)

Arguments

- **eLog**: event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction. The "date" column must consist of date objects, not character strings.
- **day1**: date of beginning of first period. Must be a date object.
- **day2**: date of end of first period. Must be a date object.
- **day3**: date of beginning of second period. Must be a date object.
- **day4**: date of third period. Must be a date object.

Details

The four date parameters must be in ascending order.

Value

- **list**: the event log with all eLog$date entries between day1 and day2
- **eLog1**: the event with all eLog$date entries between day3 and day4
- **eLog3**: eLog1 combined with eLog2, with all dates from eLog2 reduced by the time removed between eLog1 and eLog2
Examples

```
elog <- dc.ReadLines(system.file("data/cdnowElog.csv", package="BTYD"), 2:3:5)
elog[, "date"] <- as.Date(elog[, "date"], "%Y-%m-d")

# Use the cdnow data to return a 6 month event log for January, February, March, October, November, December.
period.one.start <- as.Date("1997-01-01")
period.one.end <- as.Date("1997-06-30")
period.two.start <- as.Date("1997-10-01")
period.two.end <- as.Date("1997-12-31")
reduced.elog <- dc.RemoveTimeBetween(elog, period.one.start, period.one.end,
                                      period.two.start, period.two.end)

# Note that the new elog will go up to June 30 at a maximum, since we are only using 6 months of data starting on January 1
max(reduced.elog$elog$date) # "1997-06-30"
```

---

dc.SplitUpElogForRepeatTrans

*Split Up Event Log for Repeat Transactions*

---

Description

Turns an event log into a repeat transaction event log, removing customers' first transactions. Also returns a data frame with information about customers' first and last transactions.

Usage

dc.SplitUpElogForRepeatTrans(elog)

Arguments

elog event log, which is a data frame with columns for customer ID ("cust"), date ("date"), and optionally other columns such as "sales". Each row represents an event, such as a transaction. The "date" column must contain date objects, not character strings or factors.

Value

list:

- **repeat.trans.elog** an event log containing only repeat transactions
- **cust.data** data frame containing the first and last transaction information for each customer
**discreteSimElog**  
*Discrete simulated annual event log data*

**Description**
Data simulated using BG/BB model assumptions. Contains annual transaction behavior for a period of 14 years, for a cohort of 10,000 customers who made their first transactions in 1970.

**Format**
A comma-delimited file representing an event log with 52432 entries. It has 2 columns: The customer’s ID and the date of the transaction in standard R date format.

**Details**
This dataset was simulated in order to illustrate certain data-conversion functions (see `dc.MakeRFmatrixCal`).

**donationsSummary**  
*Discrete donation data summary*

**Description**
This dataset contains a recency-frequency matrix capturing the discrete transaction behavior of 11,104 customers over 6 transaction opportunities, summarized as a recency-frequency matrix and a vector of annual transactions.

**Usage**
```r
data(donationsSummary)
```

**Format**
- **$rf.matrix** A matrix with 22 rows (for each possible recency-frequency combination in 6 calibration period transaction opportunities) and 4 columns: number of transactions during the calibration period (“x”), recency in the calibration period (“t.x”), number of transaction opportunities in the calibration period (“n.cal”), and number of customers with this recency-frequency combination in the calibration period (“custs”).
- **$rf.matrix.holdout** A matrix with 15 rows (for each possible recency-frequency combination in 5 holdout period transaction opportunities) and 4 columns: number of transactions during the holdout period (“x.star”), recency in the holdout period (“t.x.star”), number of transaction opportunities in the holdout period (“n.star”), and number of customers with the recency-frequency combination in the holdout period (“custs”).
- **$x.star** A vector with 22 elements, containing the number of transactions made by each calibration period recency-frequency bin in the holdout period. It is in the same order as $rf.matrix.
- **$annual.sales** A vector with 11 elements, containing the number of transactions made by all customers in each time period in both the calibration and holdout periods.
Details

Data from “a major nonprofit organization located in the midwestern United States that is funded in large part by donations from individuals. In 1995 the organization “acquired” 11,104 first-time supporters; in each of the following six years, these individuals either did or did not support the organization.”

This dataset contains, for each possible in-sample recency/frequency combination in the 1995 cohort, the number of customers and the number of transactions they made during the validation period.

Source

Data can be found online at http://www.brucehardie.com/notes/010/ (Associated Excel spreadsheet)

References


pnbd.compress.cbs

Compress Customer-by-Sufficient-Statistic Matrix

Description

Combines all customers with the same combination of recency, frequency and length of calibration period in the customer-by-sufficient-statistic matrix, and adds a fourth column labelled "custs" (with the number of customers belonging in each row).

Usage

pnbd.compress.cbs(cbs, rounding = 3)

Arguments

cbs calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.

rounding the function tries to ensure that there are similar customers by rounding the customer-by-sufficient-statistic matrix first. This parameter determines how many decimal places are left in the data. Negative numbers are allowed; see the documentation for round in the base package. As of the time of writing, that documentation states: "Rounding to a negative number of digits means rounding to a power of ten, so for example round(x, digits = -2) rounds to the nearest hundred."
Details

This function is meant to be used to speed up log-likelihood and parameter estimation functions. How much faster those function run depends on how similar customers are.

This function only takes columns "x", "t.x", and "T.cal" into account. All other columns will be added together - for example, if you have a spend column, the output's spend column will contain the total amount spent by all customers with an identical recency, frequency, and time observed.

Value

A customer-by-sufficient-statistic matrix with an additional column "custs", which contains the number of customers with each combination of recency, frequency and length of calibration period.

Examples

```
# Create a sample customer-by-sufficient-statistic matrix:
set.seed(7)
x <- sample(1:4, 10, replace = TRUE)
t.x <- sample(1:4, 10, replace = TRUE)
T.cal <- rep(4, 10)
ave.spend <- sample(10:20, 10, replace = TRUE)
cbs <- cbind(x, t.x, T.cal, ave.spend)
cbs

# If cbs is printed, you would note that the following
# sets of rows have the same x, t.x and T.cal:
# (1, 6, 8); (3, 9)

pnbd.compress.cbs(cbs, 0)  # No rounding necessary

# Note that all additional columns (in this case, ave.spend)
# are aggregated by sum.
```
**Arguments**

- **params**
  - Pareto/NBD parameters - a vector with \(r\), alpha, \(s\), and \(beta\), in that order. \(r\) and alpha are unobserved parameters for the NBD transaction process. \(s\) and \(beta\) are unobserved parameters for the Pareto (exponential gamma) dropout process.

- **\(T\star\)**
  - length of time for which we are calculating the expected number of transactions.

- **\(x\)**
  - number of repeat transactions in the calibration period \(T\,cal\), or a vector of calibration period frequencies.

- **\(t\,x\)**
  - time of most recent repeat transaction, or a vector of recencies.

- **\(T\,cal\)**
  - length of calibration period, or a vector of calibration period lengths.

**Details**

\[
E[X(T\,cal, T\,cal + T\star) \mid x, t.x, r, alpha, s, beta]
\]

\(T\star, x, t.x, \) and \(T\,cal\) may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

**Value**

Number of transactions a customer is expected to make in a time period of length \(t\), conditional on their past behavior. If any of the input parameters has a length greater than 1, this will be a vector of expected number of transactions.

**References**


**See Also**

- \(pnbd\,\text{Expectation}\)

**Examples**

```r
c(0.55, 10.56, 0.61, 11.64)
# Number of transactions a customer is expected to make in 2 time intervals, given that they made 10 repeat transactions in a time period # of 39 intervals, with the 10th repeat transaction occurring in the 35th interval.
pnbd\,\text{ConditionalExpectedTransactions}(params, T\star=2, x=10, t.x=35, T\,cal=39)

# We can also compare expected transactions across different calibration period behaviors:
pnbd\,\text{ConditionalExpectedTransactions}(params, T\star=2, x=5:20, t.x=25, T\,cal=39)
```
Description

Calculates the discounted expected residual transactions of a customer, given their behavior during the calibration period.

Usage

\[ \text{pnbd.DERT}(\text{params}, \ x, \ t.x, \ T.tc, \ d) \]

Arguments

- **params**: Pareto/NBD parameters - a vector with \( r \), \( \alpha \), \( s \), and \( \beta \), in that order. \( r \) and \( \alpha \) are unobserved parameters for the NBD transaction process. \( s \) and \( \beta \) are unobserved parameters for the Pareto (exponential gamma) dropout process.
- **x**: the number of repeat transactions a customer made in the calibration period, or a vector of transaction frequencies.
- **t.x**: recency: the time of the customer’s last transaction in the calibration period, or a vector of recencies.
- **T.cal**: the length of the calibration period, or a vector of calibration period lengths. Make sure that the lengths of time periods for all parameters match.
- **d**: the discount rate to be used. Make sure that it matches up with your chosen time period (do not use an annual rate for monthly data, for example).

Details

\[ \text{DERT}(d | r, \alpha, s, \beta, X = x, t.x, Ttc) \]

\( x, t.x, T.tc \) may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.

Value

The number of discounted expected residual transactions for a customer with a particular purchase pattern during the calibration period.

References


http://www.brucehardie.com/papers.html

See equation 2.
Note that this paper refers to what this package is calling discounted expected residual transactions (DERT) simply as discounted expected transactions (DET).

Examples

```r
params <- c(0.5629966, 12.5590370, 0.4081095, 10.5148048)

# 15% compounded annually has been converted to 0.0027 compounded continuously, # as we are dealing with weekly data and not annual data.
d <- 0.0027

# calculate the discounted expected residual transactions of a customer # who made 7 transactions in a calibration period that was 77.86 # weeks long, with the last transaction occurring at the end of # the 35th week.
pnbd.DERT(params, x=7, t.x=35, T.cal=77.86, d)

# We can also use vectors to compute DERT for several customers:
pnbd.DERT(params, x=1:10, t.x = 30, T.cal=77.86, d)
```

pnbd.EstimateParameters

---

**Pareto/NBD Parameter Estimation**

**Description**

Estimates parameters for the Pareto/NBD model.

**Usage**

```r
pnbd.EstimateParameters(cal.cbs, par.start = c(1, 1, 1, 1),
max.param.value = 10000)
```

**Arguments**

- `cal.cbs` : calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period. If your data is compressed (see `pnbd.compress.cbs`), a fourth column labelled "custs" (number of customers with a specific combination of recency, frequency and length of calibration period) will make this function faster.

- `par.start` : initial Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

- `max.param.value` : the upper bound on parameters.
Details

The best-fitting parameters are determined using the \texttt{pnbd.cbs.LL} function. The sum of the log-likelihood for each customer (for a set of parameters) is maximized in order to estimate parameters.

A set of starting parameters must be provided for this method. If no parameters are provided, \((1,1,1,1)\) is used as a default. It may be useful to use starting values for \(r\) and \(s\) that represent your best guess of the heterogeneity in the buy and die rate of customers. It may be necessary to run the estimation from multiple starting points to ensure that it converges. To compare the log-likelihoods of different parameters, use \texttt{pnbd.cbs.LL}.

The lower bound on the parameters to be estimated is always zero, since Pareto/NBD parameters cannot be negative. The upper bound can be set with the \texttt{max.param.value} parameter.

This function may take some time to run.

Value

Vector of estimated parameters.

References


See Also

\texttt{pnbd.cbs.LL}

Examples

data(cdnowSummary)

cal.cbs <- cdnowSummary$cbs
# cal.cbs already has column names required by method

# starting-point parameters
startingparams <- c(0.5, 6, 0.9, 8)
# estimated parameters
est.params <- pnbd.EstimateParameters(cal.cbs, startingparams)

# log-likelihood of estimated parameters
pnbd.cbs.LL(est.params, cal.cbs)

# compare the speed and results to the following:
cal.cbs.compressed <- pnbd.compress.cbs(cal.cbs)
est.params <- pnbd.EstimateParameters(cal.cbs.compressed, startingparams)
pnbd.Expectation

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns the number of repeat transactions that a randomly chosen customer (for whom we have no prior information) is expected to make in a given time period.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>pnbd.Expectation(params, t)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td>params</td>
</tr>
<tr>
<td>t</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(X(t) \mid r, \alpha, s, \beta) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of repeat transactions a customer is expected to make in a time period of length ( t ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>References</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>See Also</th>
</tr>
</thead>
<tbody>
<tr>
<td>pnbd.ConditionalExpectedTransactions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>params &lt;- c(0.55, 10.56, 0.61, 11.64)</td>
</tr>
<tr>
<td># Number of repeat transactions a customer is expected to make in 2 time intervals.</td>
</tr>
<tr>
<td>pnbd.Expectation(params, t=2)</td>
</tr>
<tr>
<td># We can also compare expected transactions over time:</td>
</tr>
<tr>
<td>pnbd.Expectation(params, t=1:10)</td>
</tr>
</tbody>
</table>
pnbd.ExpectedCumulativeTransactions

Pareto/NBD Expected Cumulative Transactions

Description

Calculates the expected cumulative total repeat transactions by all customers for the calibration and holdout periods.

Usage

pnbd.ExpectedCumulativeTransactions(params, T.cal, T.tot, n.periods.final)

Arguments

params
Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

T.cal
a vector to represent customers’ calibration period lengths (in other words, the “T.cal” column from a customer-by-sufficient-statistic matrix).

T.tot
end of holdout period. Must be a single value, not a vector.

n.periods.final
number of time periods in the calibration and holdout periods. See details.

Details

The function automatically divides the total period up into n.periods.final time intervals. n.periods.final does not have to be in the same unit of time as the T.cal data. For example:
- if your T.cal data is in weeks, and you want cumulative transactions per week, n.periods.final would equal T.star.
- if your T.cal data is in weeks, and you want cumulative transactions per day, n.periods.final would equal T.star * 7.

The holdout period should immediately follow the calibration period. This function assume that all customers’ calibration periods end on the same date, rather than starting on the same date (thus customers’ birth periods are determined using max(T.cal) - T.cal rather than assuming that it is 0).

Value

Vector of expected cumulative total repeat transactions by all customers.

See Also

pnbd.Expectation
Examples

```r
data(cdnowSummary)

cal.cbs <- cdnowSummary$cbs
# cal.cbs already has column names required by method

params <- c(0.55, 10.56, 0.61, 11.64)

# Returns a vector containing cumulative repeat transactions for 546 days.
# All parameters are in weeks; the calibration period lasted 39 weeks
# and the holdout period another 39.
pnbd.ExpectedCumulativeTransactions(params, T.cal=cal.cbs[,"T.cal"],
T.tot=78, n.periods.final=546)
```

---

**pnbd.cbs**

**Pareto/NBD Log-Likelihood**

**Description**

Calculates the log-likelihood of the Pareto/NBD model.

**Usage**

```r
pnbd.cbs.LL(params, cal.cbs)

pnbd.LL(params, x, t.x, T.cal)
```

**Arguments**

- **params**  
  Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

- **cal.cbs**  
  calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer's last transaction, not the time between the customer's last transaction and the end of the calibration period. If your data is compressed (see `pnbd.compress.cbs`), a fourth column labelled "custs" (number of customers with a specific combination of recency, frequency and length of calibration period) will make this function faster.

- **x**  
  number of repeat transactions in the calibration period T.cal, or a vector of transaction frequencies.

- **t.x**  
  time of most recent repeat transaction, or a vector of recencies.

- **T.cal**  
  length of calibration period, or a vector of calibration period lengths.
Details

x, t.x, T.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be also be a vector.

Value

For pnbd.cbs. LL, the total log-likelihood of the provided data. For pnbd. LL, a vector of log-likelihoods as long as the longest input vector (x, t.x, or T.cal).

References


See Also

pnbd.EstimateParameters

Examples

data(cdnowSummary)

cal.cbs <- cdnowSummary$cb
# cal.cbs already has column names required by method

# random assignment of parameters
params <- c(0.5, 8, 0.7, 10)
# returns the log-likelihood of the given parameters
pnbd.cbs. LL (params, cal.cbs)

# compare the speed and results to the following:
cal.cbs.compressed <- pnbd.compress.cbs(cal.cbs)
pnbd.cbs. LL (params, cal.cbs.compressed)

# Returns the log likelihood of the parameters for a customer who # made 3 transactions in a calibration period that ended at t=6, # with the last transaction occurring at t=4.
pnbd. LL (params, x=3, t.x=4, T.cal=6)

# We can also give vectors as function parameters:
set.seed(7)
x <- sample(1:4, 10, replace = TRUE)
t.x <- sample(1:4, 10, replace = TRUE)
T.cal <- rep(4, 10)
pnbd. LL (params, x, t.x, T.cal)
Pnbd.PAlive  Pareto/NBD P(Alive)

Description

Uses Pareto/NBD model parameters and a customer’s past transaction behavior to return the probability that they are still alive at the end of the calibration period.

Usage

```
pnbd.PAlive(params, x, t.x, T.cal)
```

Arguments

- **params**: Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.
- **x**: number of repeat transactions in the calibration period, or a vector of transaction frequencies.
- **t.x**: time of most recent repeat transaction, or a vector of recencies.
- **T.cal**: length of calibration period, or a vector of calibration period lengths.

Details

\[
P(\text{Alive} | X=x, t.x, T.cal, r, alpha, s, beta)
\]

x, t.x, and T.cal may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Probability that the customer is still alive at the end of the calibration period. If x, t.x, and/or T.cal has a length greater than one, then this will be a vector of probabilities (containing one element matching each element of the longest input vector).

References

Examples

```r
params <- c(0.55, 10.56, 0.61, 11.64)

pnbd.PA(ive(params, x=0, t.x=0, T.cal=39)
# 0.2924031; P(Alive) of a customer who made no repeat transactions.

pnbd.PA(ive(params, x=23, t.x=39, T.cal=39)
# 1; P(Alive) of a customer who has the same recency and total
# time observed.

pnbd.PA(ive(params, x=5:20, t.x=30, T.cal=39)
# Note the "increasing frequency paradox".

# To visualize the distribution of P(Alive) across customers:

data(cdnwSummary)
cbs <- cdnowSummary$cbs
params <- pnbd.EstimateParameters(cbs)
p.alives <- pnbd.PA(ive(params, cbs[,"x"], cbs[,"t.x"], cbs[,"T.cal"])
plot(density(p.alives))
```

Description

Plots discounted expected residual transactions for different combinations of calibration period frequency and recency.

Usage

```r
pnbd.Plot.DERT(params, x, t.x, T.cal, d, type = "wireframe")
```

Arguments

- **params**: Pareto/NBD parameters - a vector with \( r, \alpha, s, \) and \( \beta \), in that order. \( r \) and \( \alpha \) are unobserved parameters for the NBD transaction process. \( s \) and \( \beta \) are unobserved parameters for the Pareto (exponential gamma) dropout process.
- **x**: a vector of calibration period transaction frequencies.
- **t.x**: a vector of recencies of transactions.
- **T.cal**: length of the calibration period - must be a single value and not a vector.
- **d**: the discount rate to be used. Make sure that it matches up with your chosen time period (do not use an annual rate for monthly data, for example).
- **type**: must be either "persp" (perspective - 3 dimensional) or "contour". Determines the type of plot produced by this function.
Details

A matrix with DERT values for all the possible combinations of the given \( x \) and \( t.x \).

Value

A matrix with discounted expected residual transaction values for every combination of calibration period frequency and calibration period recency.

References


Note that this paper refers to what this package is calling discounted expected residual transactions (DERT) simply as discounted expected transactions (DET).

Examples

```
# The RFM and CLV paper uses all 78 weeks of the cdnow data to
# estimate parameters. These parameters can be estimated as follows:

# elog <- dc.ReadLines(system.file("data/cdnowElog.csv", package="BTYD"),2,3)
# cal.cbs <- dc.ElogToCbsCbt(elog)$cal$cbs
# pnbd.EstimateParameters(cal.cbs)
# (The final function was run several times with its own output as
# input for starting parameters, to ensure that the result converged).

params <- c(0.5629966, 12.5590370, 0.4081095, 10.5148048)

# 15% compounded annually has been converted to 0.0027 compounded continously,
# as we are dealing with weekly data and not annual data.
# d <- 0.0027

pnbd.Plot.DERT(params, x=0:14, t.x=0:77, T.cal=77.86, d, type="persp")

pnbd.Plot.DERT(params, x=0:14, t.x=0:77, T.cal=77.86, d, type="contour")
```
**Arguments**

```r
params
```
Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

```r
lim
```
upper-bound of the x-axis. A number is chosen by the function if none is provided.

**Details**

This returns the distribution of each customer's exponential parameter that determines their lifetime (using the Pareto/NBD assumption that a customer's lifetime can be modeled with an exponential distribution).

**Value**

Distribution of customers' propensities to drop out.

**Examples**

```r
params <- c(0.55, 10.56, 0.61, 11.64)
pnbd.PlotDropoutRateHeterogeneity(params)
params <- c(0.55, 10.56, 3, 11.64)
pnbd.PlotDropoutRateHeterogeneity(params)
```

---

**pnbd.PlotFrequencyInCalibration**

*Pareto/NBD Plot Frequency in Calibration Period*

**Description**

Plots a histogram and returns a matrix comparing the actual and expected number of customers who made a certain number of repeat transactions in the calibration period, binned according to calibration period frequencies.

**Usage**

```r
pnbd.PlotFrequencyInCalibration(params, cal.cbs, censor, plotZero = TRUE,
                                xlab = "Calibration period transactions", ylab = "Customers",
                                title = "Frequency of Repeat Transactions")
```

**Arguments**

```r
params
```
Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

```r
cal.cbs
```
calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x") and total time observed ("T.cal").
censor integer used to censor the data. See details.
plotZero If FALSE, the histogram will exclude the zero bin.
xlab descriptive label for the x axis.
ylab descriptive label for the y axis.
title title placed on the top-center of the plot.

Details
This function requires a censor number, which cannot be higher than the highest frequency in the calibration period CBS. The output matrix will have (censor + 1) bins, starting at frequencies of 0 transactions and ending at a bin representing calibration period frequencies at or greater than the censor number. The plot may or may not include a bin for zero frequencies, depending on the plotZero parameter.

Value
Calibration period repeat transaction frequency comparison matrix (actual vs. expected).

Examples
data(cdnnowSummary)
cal.cbs <- cdnowSummary$CBS  # cal.cbs already has column names required by method

# parameters estimated using pnbd.EstimateParameters
est.params <- cdnowSummary$est.params
# the maximum censor number that can be used
max(cal.cbs[, "x"])
pnbd.PlotFrequencyInCalibration(est.params, cal.cbs, censor=7)
Arguments

params: Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

T.star: length of the holdout period.

cal.cbs: calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer's last transaction, not the time between the customer's last transaction and the end of the calibration period.

x.star: vector of transactions made by each customer in the holdout period.

censor: integer used to censor the data. See details.

xlab: descriptive label for the x axis.

ylab: descriptive label for the y axis.

xticklab: vector containing a label for each tick mark on the x axis.

title: title placed on the top-center of the plot.

Details

This function requires a censor number, which cannot be higher than the highest frequency in the calibration period CBS. The output matrix will have (censor + 1) bins, starting at frequencies of 0 transactions and ending at a bin representing calibration period frequencies at or greater than the censor number.

Value

Holdout period transaction frequency comparison matrix (actual vs. expected).

Examples

```r
data(cdnowSummary)

cal.cbs <- cdnowSummary$CBS
# cal.cbs already has column names required by method

# number of transactions by each customer in the 39 weeks
# following the calibration period
x.star <- cal.cbs[,"x.star"]

# parameters estimated using pnbd.EstimateParameters
est.params <- cdnowSummary$est.params
# the maximum censor number that can be used
max(cal.cbs[,"x"])

# plot conditional expected holdout period frequencies,
# binned according to calibration period frequencies
pnbd.PlotFreqVsConditionalExpectedFrequency(est.params, T.star=39, cal.cbs, x.star, censor=7)
```
Description

Plots the actual and conditional expected number of transactions made by customers in the holdout period, binned according to calibration period recencies. Also returns a matrix with this comparison and the number of customers in each bin.

Usage

```r
pnbd.PlotRecVsConditionalExpectedFrequency(params, cal.cbs, T.star, x.star, 
xlab = "Calibration period recency", ylab = "Holdout period transactions", 
xticklab = NULL, 
title = "Actual vs. Conditional Expected Transactions by Recency")
```

Arguments

- `params`: Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.
- `cal.cbs`: calibration period CBS (customer by sufficient statistic). It must contain columns for frequency ("x"), recency ("t.x"), and total time observed ("T.cal"). Note that recency must be the time between the start of the calibration period and the customer’s last transaction, not the time between the customer’s last transaction and the end of the calibration period.
- `T.star`: length of then holdout period.
- `x.star`: vector of transactions made by each customer in the holdout period.
- `xlab`: descriptive label for the x axis.
- `ylab`: descriptive label for the y axis.
- `xticklab`: vector containing a label for each tick mark on the x axis.
- `title`: title placed on the top-center of the plot.

Details

This function does bin customers exactly according to recency; it bins customers according to integer units of the time period of cal.cbs. Therefore, if you are using weeks in your data, customers will be binned as follows: customers with recencies between the start of the calibration period (inclusive) and the end of week one (exclusive); customers with recencies between the end of week one (inclusive) and the end of week two (exclusive); etc.

The matrix and plot will contain the actual number of transactions made by each bin in the holdout period, as well as the expected number of transactions made by that bin in the holdout period, conditional on that bin’s behavior during the calibration period.
Value

Matrix comparing actual and conditional expected transactions in the holdout period.

Examples

```r
data(cdnnowSummary)
cal.cbs <- cdnowSummary$cbs
# cal.cbs already has column names required by method

# number of transactions by each customer in the 39 weeks following
# the calibration period
x.star <- cal.cbs[,'x.star']

# parameters estimated using pnbd.EstimateParameters
est.params <- cdnowSummary$est.params

# plot conditional expected holdout period transactions, binned according to
# calibration period recencies
pnbd.PlotRecVsConditionalExpectedFrequency(est.params, cal.cbs, T.star=39, x.star)
```

Description

Plots the actual and expected cumulative total repeat transactions by all customers for the calibration and holdout periods, and returns this comparison in a matrix.

Usage

```r
pnbd.PlotTrackingCum(params, T.cal, T.tot, actual.cu.tracking.data, 
  xlab = "Week", ylab = "Cumulative Transactions", xticklab = NULL, 
  title = "Tracking Cumulative Transactions")
```

Arguments

- **params**: Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.
- **T.cal**: a vector to represent customers’ calibration period lengths (in other words, the "T.cal" column from a customer-by-sufficient-statistic matrix).
- **T.tot**: end of holdout period. Must be a single value, not a vector.
- **actual.cu.tracking.data**: vector containing the cumulative number of repeat transactions made by customers for each period in the total time period (both calibration and holdout periods). See details.
pnbd.PlotTrackingInc

Description

Plots the actual and expected incremental total repeat transactions by all customers for the calibration and holdout periods, and returns this comparison in a matrix.
Usage

```r
pnbd.PlotTrackingInc(params, T.cal, T.tot, actual.inc.tracking.data,
  xlab = "Week", ylab = "Transactions", xticklab = NULL,
  title = "Tracking Weekly Transactions")
```

Arguments

- **params**: Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.
- **T.cal**: a vector to represent customers’ calibration period lengths (in other words, the “T.cal” column from a customer-by-sufficient-statistic matrix).
- **T.tot**: end of holdout period. Must be a single value, not a vector.
- **actual.inc.tracking.data**: vector containing the incremental number of repeat transactions made by customers for each period in the total time period (both calibration and holdout periods). See details.
- **xlab**: descriptive label for the x axis.
- **ylab**: descriptive label for the y axis.
- **xticklab**: vector containing a label for each tick mark on the x axis.
- **title**: title placed on the top-center of the plot.

Details

`actual.inc.tracking.data` does not have to be in the same unit of time as the `T.cal` data. `T.tot` will automatically be divided into periods to match the length of `actual.inc.tracking.data`. See `pnbd.ExpectedCumulativeTransactions`.

The holdout period should immediately follow the calibration period. This function assume that all customers’ calibration periods end on the same date, rather than starting on the same date (thus customers’ birth periods are determined using `max(T.cal) - T.cal` rather than assuming that it is 0).

Value

Matrix containing actual and expected incremental repeat transactions.

Examples

```r
data(cdnowSummary)
cal.cbs <- cdnowSummary$cbs
# cal.cbs already has column names required by method

# Cumulative repeat transactions made by all customers across calibration
# and holdout periods
cu.tracking <- cdnowSummary$cu.tracking
# make the tracking data incremental
inc.tracking <- dc.CumulativeToIncremental(cu.tracking)
```
Description

Plots and returns the estimated gamma distribution of lambda (customers’ propensities to purchase).

Usage

pnbd.PlotTransactionRateHeterogeneity(params, lim = NULL)

Arguments

params  Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

lim  upper-bound of the x-axis. A number is chosen by the function if none is provided.

Details

This returns the distribution of each customer's Poisson parameter, which determines the level of their purchasing (using the Pareto/NBD assumption that purchasing on the individual level can be modeled with a Poisson distribution).

Value

Distribution of customers' propensities to purchase.

Examples

params <- c(0.55, 10.56, 0.61, 11.64)

pnbd.PlotTransactionRateHeterogeneity(params)

params <- c(3, 10.56, 0.61, 11.64)

pnbd.PlotTransactionRateHeterogeneity(params)
Pareto/NBD Probability Mass Function

Description

Probability mass function for the Pareto/NBD.

Usage

\[ \text{pnbd.pmf}(\text{params, } t, x) \]

Arguments

- \text{params} \hspace{1cm} \text{Pareto/NBD parameters - a vector with } r, \alpha, s, \text{ and beta, in that order. } r \text{ and alpha are unobserved parameters for the NBD transaction process. } s \text{ and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.}
- \text{t} \hspace{1cm} \text{length end of time period for which probability is being computed. May also be a vector.}
- \text{x} \hspace{1cm} \text{number of repeat transactions by a random customer in the period defined by t. May also be a vector.}

Details

\[ P(X(t)=x \mid r, \alpha, s, \beta) \]. Returns the probability that a customer makes \( x \) repeat transactions in the time interval \((0, t]\).

Parameters \( t \) and \( x \) may be vectors. The standard rules for vector operations apply - if they are not of the same length, the shorter vector will be recycled (start over at the first element) until it is as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

Value

Probability of \( X(t)=x \) conditional on model parameters. If \( t \) and/or \( x \) has a length greater than one, a vector of probabilities will be returned.

References

Examples

```r
params <- c(0.55, 10.56, 0.61, 11.64)
# probability that a customer will make 10 repeat transactions in the
# time interval (0,2]
pnbd.pmf(params, t=2, x=10)
# probability that a customer will make no repeat transactions in the
# time interval (0,39]
pnbd.pmf(params, t=39, x=0)

# Vectors may also be used as arguments:
pnbd.pmf(params, t=30, x=11:20)
```

**pnbd.pmf General**  
*Generalized Pareto/NBD Probability Mass Function*

**Description**

Generalized probability mass function for the Pareto/NBD.

**Usage**

```r
pnbd.pmf General(params, t.start, t.end, x)
```

**Arguments**

- `params`:
  Pareto/NBD parameters - a vector with r, alpha, s, and beta, in that order. r and alpha are unobserved parameters for the NBD transaction process. s and beta are unobserved parameters for the Pareto (exponential gamma) dropout process.

- `t.start`:
  Start of time period for which probability is being calculated. It can also be a vector of values.

- `t.end`:
  End of time period for which probability is being calculated. It can also be a vector of values.

- `x`:
  Number of repeat transactions by a random customer in the period defined by (t.start, t.end]. It can also be a vector of values.

**Details**

\[ P(X(t.start, t.end)=x \mid r, \alpha, s, \beta) \]

Returns the probability that a customer makes \(x\) repeat transactions in the time interval \((t.start, t.end]\).

It is impossible for a customer to make a negative number of repeat transactions. This function will return an error if it is given negative times or a negative number of repeat transactions. This function will also return an error if \(t.end\) is less than \(t.start\).

\(t.start\), \(t.end\), and \(x\) may be vectors. The standard rules for vector operations apply - if they are not of the same length, shorter vectors will be recycled (start over at the first element) until they are as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.
spend.estimateparameters

Value

Probability of x transaction occurring between t.start and t.end conditional on model parameters. If t.start, t.end, and/or x has a length greater than one, a vector of probabilities will be returned.

References


Examples

```r
params <- c(0.55, 10.56, 0.61, 11.64)
# probability that a customer will make 10 repeat transactions in the
# time interval [1,2]
pnbd.pmf.General(params, t.start=1, t.end=2, x=10)
# probability that a customer will make no repeat transactions in the
# time interval (39,78]
pnbd.pmf.General(params, t.start=39, t.end=78, x=0)
```

Description

Estimates parameters for the gamma-gamma spend model.

Usage

```r
spend.estimateparameters(m.x.vector, x.vector, par.start = c(1, 1, 1),
                         max.param.value = 10000)
```

Arguments

- `m.x.vector`: a vector with each customer’s average observed transaction value in the calibration period.
- `x.vector`: a vector with the number of transactions each customer made in the calibration period. Must correspond to m.x.vector in terms of ordering of customers and length of the vector.
- `par.start`: initial vector of gamma-gamma parameters: p, q, and gamma, in that order. p is the shape parameter for each transaction. The scale parameter for each transaction is distributed across customers according to a gamma distribution with parameters q (shape) and gamma (scale).
- `max.param.value`: the upper bound on parameters.
Details

The best-fitting parameters are determined using the `spend.LL` function. The sum of the log-likelihood for each customer (for a set of parameters) is maximized in order to estimate parameters.

A set of starting parameters must be provided for this method. If no parameters are provided, (1,1,1,1) is used as a default. It may be necessary to run the estimation from multiple starting points to ensure that it converges. To compare the log-likelihoods of different parameters, use `spend.LL`.

The lower bound on the parameters to be estimated is always zero, since gamma-gamma parameters cannot be negative. The upper bound can be set with the `max.param.value` parameter.

Value

Vector of estimated parameters.

Examples

data(cdnowSummary)
ave.spend <- cdnowSummary$m.x
tot.trans <- cdnowSummary$cb$s[,"x"]

# There will be many warnings due to the zeroes that are included in the data above. To avoid them, use the following:
# (see example for `spend.LL`)
ave.spend <- ave.spend[which(tot.trans > 0)]
tot.trans <- tot.trans[which(tot.trans > 0)]

# We will let the spend function use default starting parameters
spend.EstimateParameters(ave.spend, tot.trans)

spend.expected.value   Conditional expected transaction value

Description

Calculates the expected transaction value for a customer, conditional on the number of transaction and average transaction value during the calibration period.

Usage

`spend.expected.value(params, m.x, x)`

Arguments

| params          | a vector of gamma-gamma parameters: p, q, and gamma, in that order. p is the shape parameter for each transaction. The scale parameter for each transaction is distributed across customers according to a gamma distribution with parameters q (shape) and gamma (scale). |
the customer’s average observed transaction value in the calibration period. May also be a vector of average observed transaction values - see details.

the number of transactions the customer made in the calibration period. May also be a vector of frequencies - see details.

**Details**

\[ E(M \mid p, q, \gamma, m \cdot x, x) \]

\( m \cdot x \) and \( x \) may be vectors. The standard rules for vector operations apply - if they are not of the same length, the shorter vector will be recycled (start over at the first element) until it is as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

**Value**

The expected transaction value for a customer conditional on their transaction behavior during the calibration period. If \( m \cdot x \) or \( x \) has a length greater than one, then a vector of expected transaction values will be returned.

**References**


**Examples**

```r
params <- c(6, 4, 16)
# calculate the expected transaction value of a customer
# who spent an average of $35 over 3 transactions.
spend.expected.value(params, m.x=35, x=3)

# m.x and x may be vectors:
spend.expected.value(params, m.x=30:40, x=3)
spend.expected.value(params, m.x=35, x=1:10)
spend.expected.value(params, m.x=30:40, x=1:11)
```

**Description**

Calculates the log-likelihood of the gamma-gamma model for customer spending.

**Usage**

`spend.LL(params, m.x, x)`
Arguments

params a vector of gamma-gamma parameters: p, q, and gamma, in that order. p is the shape parameter for each transaction. The scale parameter for each transaction is distributed across customers according to a gamma distribution with parameters q (shape) and gamma (scale).

m.x a customer’s average observed transaction value in the calibration period, or a vector of average observed transaction values - see details.

x the number of transactions a customer made in the calibration period or a vector of frequencies - see details.

Details

m.x and x may be vectors. The standard rules for vector operations apply - if they are not of the same length, the shorter vector will be recycled (start over at the first element) until it is as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of log-likelihoods.

Value

The log-likelihood of the gamma-gamma model. If m.x or x has a length greater than 1, this is a vector of log-likelihoods.

References


Examples

data(cdnowSummary)
ave.spend <- cdnowSummary$m.x;
tot.trans <- cdnowSummary$cbs[,"x"];
params <- c(6.25, 3.74, 15.44);

# get the total log-likelihood of the data and parameters
# above. There will be many warnings due to the zeroes that are
# included in the data. If you wish to avoid these warnings, use:

# ave.spend <- ave.spend[which(tot.trans > 0)];
# and
# tot.trans <- tot.trans[which(tot.trans > 0)];

# Note that we used tot.trans to remove the zeroes from ave.spend.
# This is because we need the vectors to be the same length, and it
# is possible that your data include customers who made transactions
# worth zero dollars (in which case the vector lengths would differ
# if we used ave.spend to remove the zeroes from ave.spend).
spend.marginal.likelihood

\[
\text{sum(spend.LL(params, ave.spend, tot.trans))}
\]

# This log-likelihood may be different than mentioned in the
# referenced paper; in the paper, a slightly different function
# which relies on total spend (not average spend) is used.

spend.marginal.likelihood

Gamma-gamma marginal likelihood

Description

Calculates the marginal likelihood of a customer's average transaction value.

Usage

spend.marginal.likelihood(params, m.x, x)

Arguments

params a vector of gamma-gamma parameters: p, q, and gamma, in that order. p is the shape parameter for each transaction. The scale parameter for each transaction is distributed across customers according to a gamma distribution with parameters q (shape) and gamma (scale).

m.x the customer's average observed transaction value in the calibration period. May also be a vector of average observed transaction values - see details.

x the number of transactions the customer made in the calibration period. May also be a vector of frequencies - see details.

Details

m.x and x may be vectors. The standard rules for vector operations apply - if they are not of the same length, the shorter vector will be recycled (start over at the first element) until it is as long as the longest vector. It is advisable to keep vectors to the same length and to use single values for parameters that are to be the same for all calculations. If one of these parameters has a length greater than one, the output will be a vector of probabilities.

This function will issue a warning if any of m.x or x is 0, and will return a marginal likelihood of 0 for those values.

\[ f(m.x \mid p, q, gamma, x) \]

Value

The marginal distribution of a customer's average transaction value. If m.x or x has a length greater than 1, a vector of marginal likelihoods will be returned.
References
http://www.brucehardie.com/papers.html
See equation 3.

Examples
params <- c(6, 4, 16)
# calculate the marginal distribution of the average transaction value
# of a customer who spent an average of $35 over 3 transactions.
spend.marginal.likelihood(params, m.x=35, x=3)

# Several values can also be computed at once:
spend.marginal.likelihood(params, m.x=30:40, x=3)
spend.marginal.likelihood(params, m.x=35, x=1:10)
spend.marginal.likelihood(params, m.x=30:40, x=1:11)

spend.plot.average.transaction.value

Plot Actual vs. Expected Average Transaction Value

Description
Plots the actual and expected densities of average transaction values, and returns a vector with each
customer’s average transaction value probability.

Usage
spend.plot.average.transaction.value(params, m.x.vector, x.vector,
xlab = "Average Transaction Value",
ylab = "Marginal Distribution of Average Transaction Value",
title = "Actual vs. Expected Average Transaction Value Across Customers")

Arguments
params a vector of gamma-gamma parameters: p, q, and gamma, in that order. p is the
shape parameter for each transaction. The scale parameter for each transaction is
distributed across customers according to a gamma distribution with parameters
q (shape) and gamma (scale).
m.x.vector a vector with each customer’s average observed transaction value in the calibra-
tion period.
x.vector a vector with the number of transactions each customer made in the calibration
period. Must correspond to m.x.vector in terms of ordering of customers and
length of the vector.
xlab descriptive label for the x axis.
ylab descriptive label for the y axis.
title title placed on the top-center of the plot.
spend.plot.average.transaction.value

Value

a vector with the probability of each customer’s average transaction value.

See Also

spend.marginal.likelihood

Examples

data(cnowSummary)
ave.spend <- cnowSummary$m.x
tot.trans <- cnowSummary$cbs[,"x"]
params <- c(6.25, 3.74, 15.44)

# Plot the actual and expected average transaction value across customers.
f.m.x <- spend.plot.average.transaction.value(params, ave.spend, tot.trans)
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